Point-Sampled Shape Representations

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Why Point-Based Graphics ?

- simplicity
- generality
- flexibility

- efficiency ?
  – point- vs. splat-approximation
  – GPU processing
- quality ?

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You are here ...

- 1.) Science
- ...
  – 1.1.) Computer Science
  – ...
    - 1.1.1.) Computer Graphics
    - ...
      – 1.1.1.1.) Point-based Graphics

        1.1.1.1.1.) Point-based Rendering Technology
        1.1.1.1.2.) Point-based Geometry Processing

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Overview

- point-based representations
  – shape approximation
  – surface topology
- octree point clouds
- optimized splat subsampling

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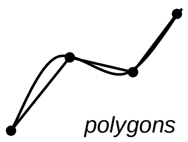
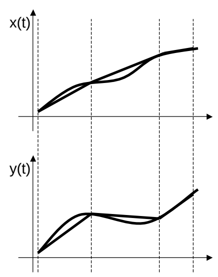
Point-Based Approximation

- what is the approximation power ?

error = $O(h^2)$



polygons



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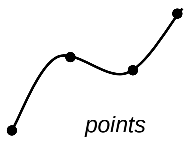
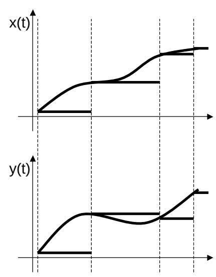


points



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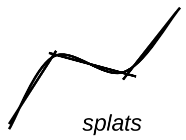
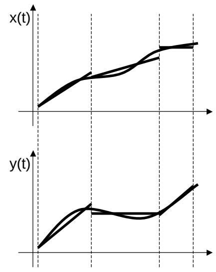
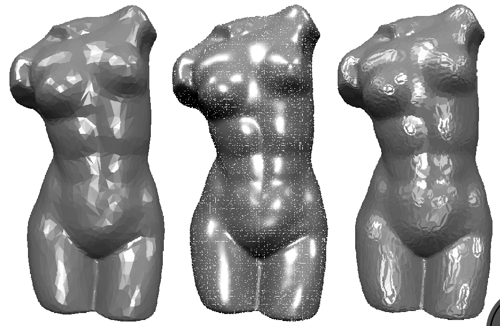


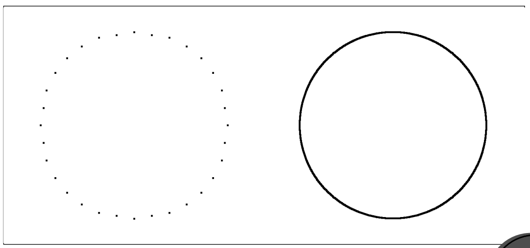


Point-Based Approximation



Point-Based Approximation

- what is the required precision ?



Point-Based Approximation

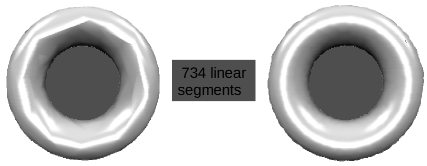
- what is the required precision ?
  - *Points* : precision = O(sampling density)
    number = O(surface area)
  - *Splats* : precision = O(sampling density ^{2})
    number = O(surface curvature)

Consequences

- pure point-based representations
  - insufficient object space approximation power
  - screen-space dependent sampling resolution
  - screen-space dependent sampling resolution
  - forward mapping techniques independent from scene complexity ?!
  - efficient culling and adaptive super-sampling techniques required

Consequences

- splat-based representations are (as least) as powerful as polygon meshes
  - locally optimal linear approximation (ellipses)
  - added flexibility („ C^{-1} “)
  - sharp features can be represented (splat clipping)



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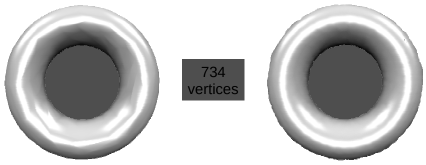

Point-Based Surface Topology

- manifold surfaces are at least C^0
- locally independent approximation yields C^{-1}
- visual *continuity* through overlapping splats (object vs. image space)
- visual *smoothness* through normal blending
- topology information embedded in a point cloud ?

Point-Based Surface Topology

- ε-neighborhood
  - symmetric, non-manifold, uniform
  - super-linear complexity
- uniform sampling vs. „ *r* - sampling “
  - geometrical precision
  - topological precision
- k-nearest neighborhood
  - asymmetric, non-manifold, adaptive
  - linear complexity

Overview

- point-based representations
  - shape approximation
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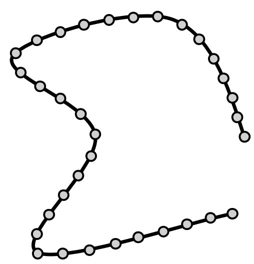
- octree point clouds
- optimized splat subsampling

Point Clouds

- piecewise constant approximation
  - sampling resolution : h
  - O(h^2) sample points
  - 3 * log(h) bits per sample
  - total complexity O(h^2 * log(h))
  - can we obtain O(h^2) total complexity ?

Point Clouds



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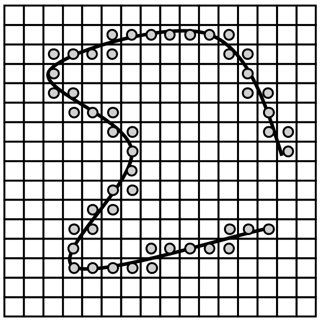

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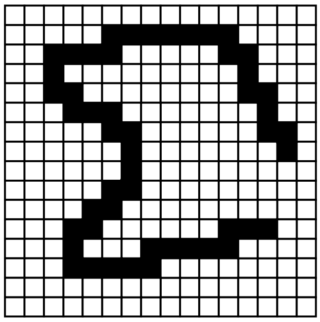
Point Clouds



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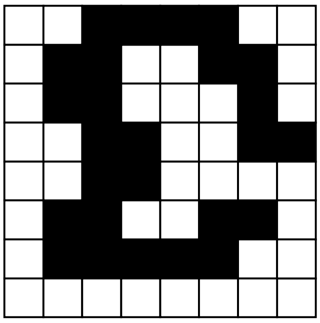
Octree Point Clouds



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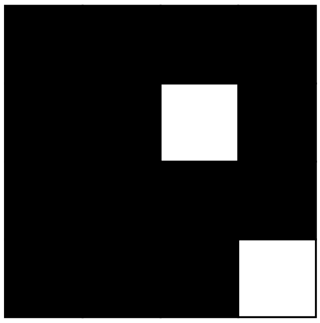
Octree Point Clouds



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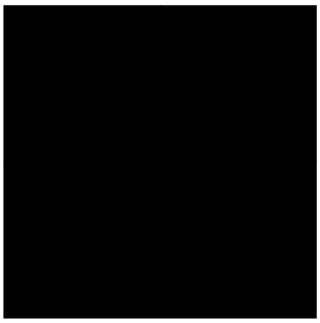
Octree Point Clouds



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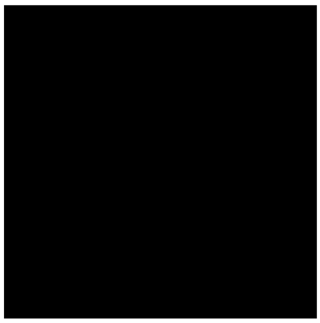

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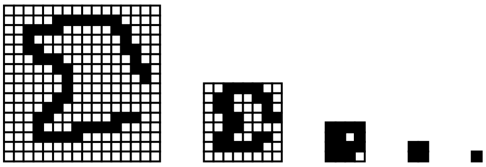
Zero Tree Coding



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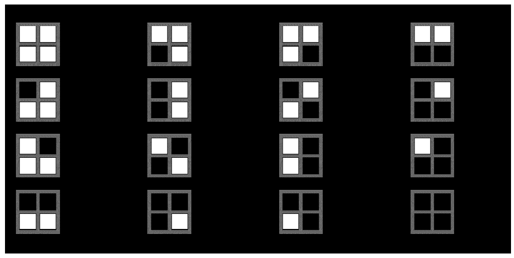

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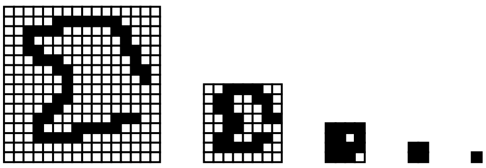

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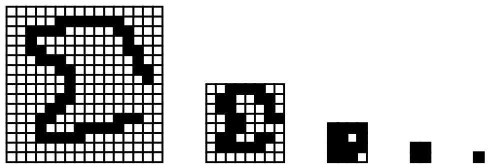

O(h^3)

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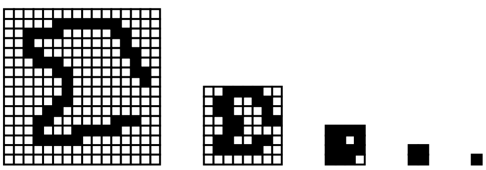

O(h^2)

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Octree Point Clouds

- storage per point
  - 8/4 + 8/16 + ... = 8/3 = 2.67 bit *(uncompressed)*
  - 1.00 – 1.50 bit *(entropy encoded)*
- resolution independent : O(h^2)
  - coarser octree levels encode *many* samples
- fast rendering by octree traversal
  - 4 scalar additions and 2 divisions per point
- level of detail representation

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Octree Traversal

- fixed translation vectors for cell centers

Level i   Level i-1

$$d_{i,j} = 2^{i-2}h \begin{pmatrix} \pm 1 \\ \pm 1 \\ \pm 1 \end{pmatrix}$$

Octree Traversal

- leaf node centers

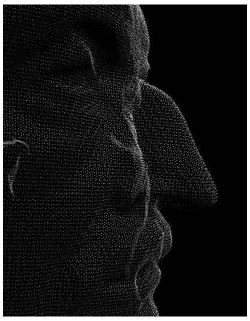
$$p = c + \sum_{i=1}^{k} d_{i,j_i}$$

- modelview + viewport transformation

$$Mp = Mc + \sum_{i=1}^{k} Md_{i,j_i} = c' + \sum_{i=1}^{k} d'_{i,j_i}$$

- incremental summation during traversal

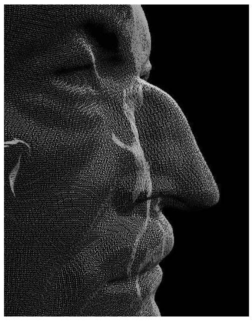
Level of Detail



8 octree levels

compression factor ≈ 1:24

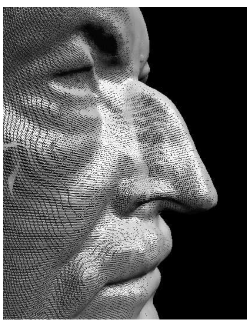
Level of Detail



9 octree levels

compression factor ≈ 1:27

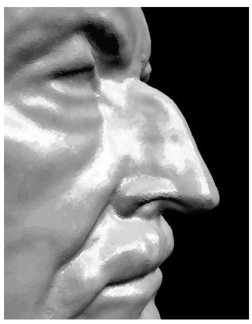
Level of Detail



10 octree levels

compression factor ≈ 1:30

Level of Detail



11 octree levels

compression factor ≈ 1:33

Level of Detail

Level of Detail

Level of Detail

Level of Detail

Progressive Transmission

Overview

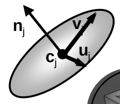
- point-based representations
  - shape approximation
  - surface topology
- octree point clouds
- optimized splat subsampling

Problem Specification

- given:
  - sample points $\mathbf{p}_i$ on a surface
  - approximation tolerance ε

Problem Specification

- given:
  - sample points $\mathbf{p}_i$ on a surface
  - approximation tolerance ε
- find:
  - minimal set of elliptical splats $S_j = (\mathbf{c}_j, \mathbf{u}_j, \mathbf{v}_j)$
  - all samples within ε
  - no holes
  - most regular splat distribution

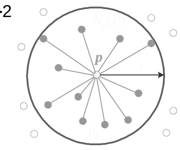


Approximation Error

- distance of a sample point to a set of splats
  (minimum projected distance)
- replace each splat by an 2ε-cylinder
- splat overlap in object space ?
  - union of solids
  - projected overlap

Surface Structure

- surface samples $\mathbf{p}_i$
- k-nearest neighbor graph N(i,j)
- estimated normals $\mathbf{n}_i$
- surface area element $\omega_i = r^2$
- splats S_j
- coverage relation C(i,j)
- surface patches P_j = C(*,j)



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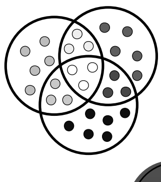
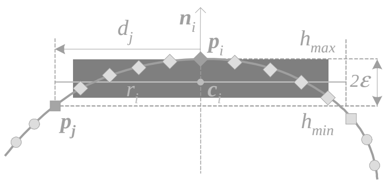


Our Approach ...

- sub-problems ...
  - global error control
  - prevent holes
  - optimal splat distribution
- techniques ...
  - one-sided Hausdorff distance (splat generation)
  - discrete coverage estimation (set operations)
  - global relaxation (better than greedy)

Splat Generation

- grow a *candidate* splat for each point $\mathbf{p}_i$
  - no least squares fitting (fixed normal, maximum deviation)



Splat Generation

- grow a *candidate* splat for each point $\mathbf{p}_i$
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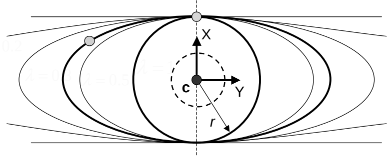
  - align elliptical splats to principal directions



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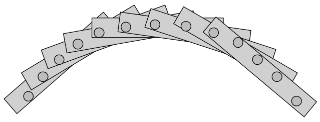
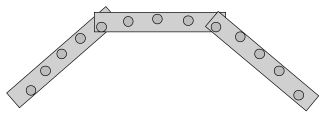


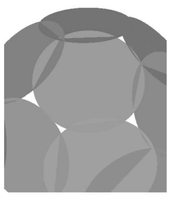
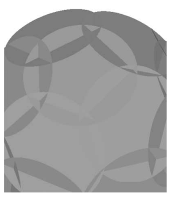
Splat Generation

- grow a *candidate* splat for each point $\mathbf{p}_i$
  - no least squares fitting (fixed normal, maximum deviation)
  - align elliptical splats to principal directions
  - each selection satisfies error threshold



Coverage Estimate

- each sample has to be assigned to a splat
- guarantee sufficient overlap

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- each sample has to be assigned to a splat
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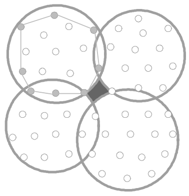


Coverage Estimate

- each sample has to be assigned to a splat
- guarantee sufficient overlap
- modified coverage relation C´(i,j)
- set operations:
  check if *active* splats cover all samples
- complexity depends on
  – number of active splats
  – number of input samples

Greedy Selection

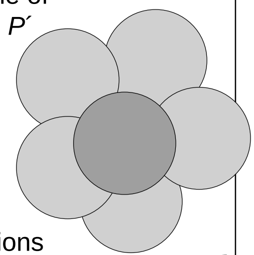
- any selection of candidates satisfies the error tolerance
- find a selection that covers all points
- greedy selection
  – largest un-covered patch

Global Relaxation

- optimize splat distribution
- two set-operations ...
  – minimize overlap
  – remove redundant splats
- preserve coverage *(local updates only)*
  – kernel of a splat $K_j \subseteq P_j$
- iterate over all splats

Minimize Overlap

- replace a splat P_j by one of its k-nearest neighbors P'
- minimize overlap with nearby active splats
- preserve full coverage (kernel K_j)
- simple local set operations



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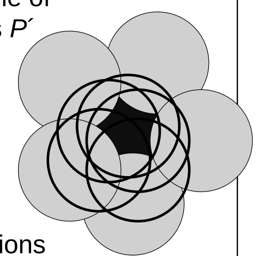

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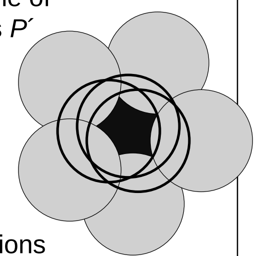

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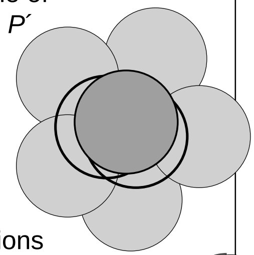

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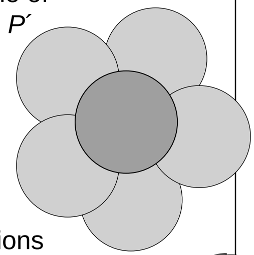

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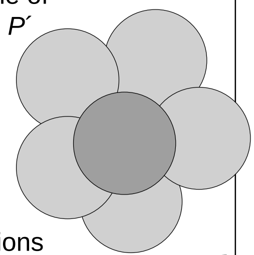
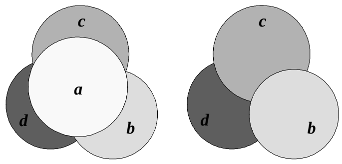


Remove Redundant Splats

- greedy selection causes redundancy
- remove and re-distribute neighboring active splats



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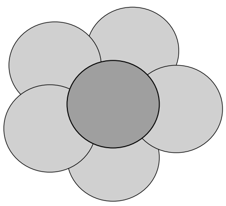

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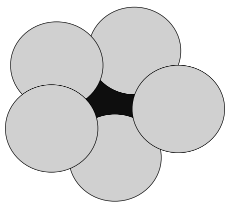

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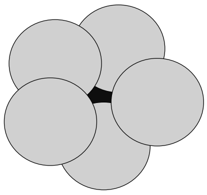

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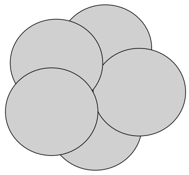

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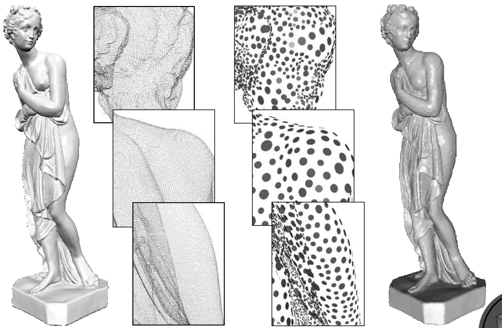
Examples



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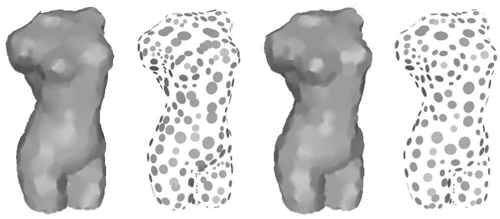
Examples

input 170K error 0.47 %



422 333

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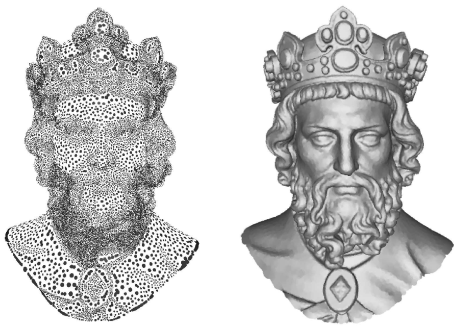
Examples



600K

66K

0.03%

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Comparison

- greedy vs. global relaxation

734, 0.29 % 493, 0.29 %

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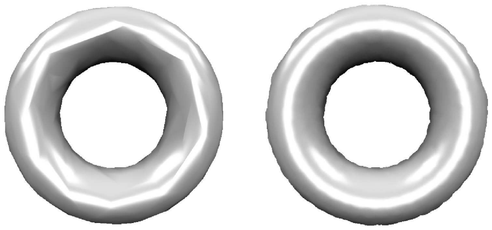
Comparison

- splats vs triangles



734 triangles 734 splats

Comparison

- circular vs. elliptical splats

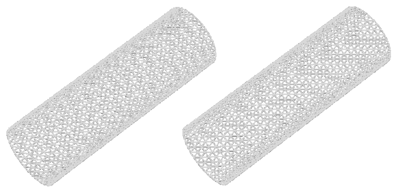
734, 0.2 % 510, 0.2 %

Advantages

- exploit full flexibility of splat represen-tations (k-nearest neighbors)
- global relaxation leads to better results than greedy selection
- take full splat geometry into account, not just the centers

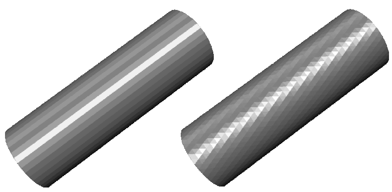
Visual Approximation Quality

- approximate normal vectors
- known problem of polygons



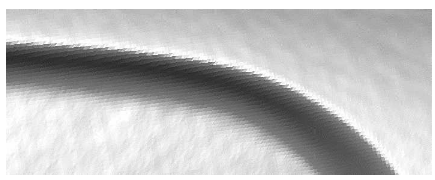
Visual Approximation Quality

- approximate normal vectors
- known problem of polygons



Visual Approximation Quality

- approximate normal vectors
- known problem of polygons



Visual Approximation Quality

- approximate normal vectors
- known problem of polygons

(where phong shading doesn‘t help)

Phong Splatting

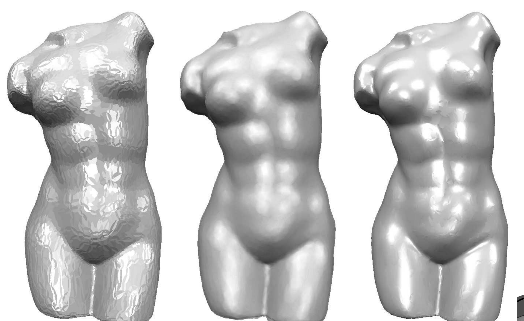
- splat $S_j = (\mathbf{c}_j, \mathbf{u}_j, \mathbf{v}_j, \mathbf{n}_j, \alpha_j, \beta_j, rgb_j)$
- $(\mathbf{c}_j, \mathbf{u}_j, \mathbf{v}_j)$ obtained by least squares
  - tangents aligned to principal directions
- $(\mathbf{n}_j, \alpha_j, \beta_j)$ obtained by least squares
  - w.r.t. splat parametrization
  - normal vector length doesn‘t matter

Examples

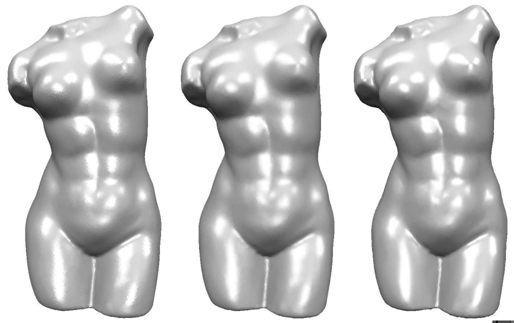
Examples

Examples

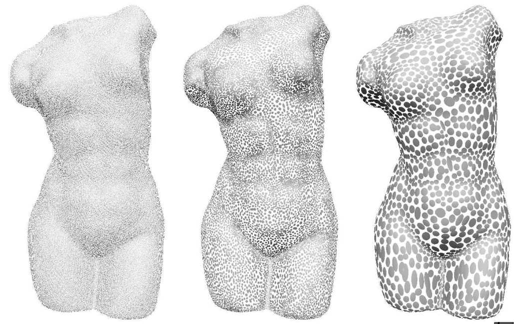
Comparison



Comparison



Comparison



Overview

- point-based representations
  - shape approximation
  - surface topology
- octree point clouds
- optimized splat subsampling

Conclusions

- point-based representations
  - good for screen space blending
  - view-independent sampling causes redundancy
  - hierarchical octree representation
- splat-based representation
- performance ???

Conclusions

- point-based representations
- splat-based representation
  - same approximation *order* as polygons
    - ellipses approximate better than triangles
    - overlap more flexible than manifold consistency
  - sharp corners
  - high quality rendering
- performance ???

Conclusions

- point-based representations
- splat-based representation
- performance ???
  - phong splatting improves visual quality and allows for sparser representations
  - why is the polygon rate still higher than the splat rate ?