- points $p : p \in \mathbb{R}^3$ $p = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{\beta} = x b_1 + y b_2 + z b_3$
- lines $\beta = \{b_1, b_2, b_3\}$
- planes

Dot product: $\langle p,q\rangle = \left\langle \begin{pmatrix}p_x\\p_y\\p_z\end{pmatrix}, \begin{pmatrix}q_x\\q_y\\q_z\end{pmatrix} \right\rangle = p_x q_x + p_y q_y + p_z q_z$

$= p^T q$

$\rightarrow$ Norms

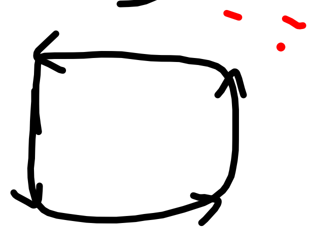
$\langle p,p\rangle = \|p\|^2$

$\langle p,q\rangle = \|p\| \cdot \|q\| \cdot \cos(\alpha)$

Cross product:

$$R = p \times q = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \times \begin{pmatrix} q_x \\ q_y \\ q_z \end{pmatrix} = \begin{pmatrix} p_y q_z - p_z q_y \\ p_z q_x - p_x q_z \\ p_x q_y - p_y q_x \end{pmatrix}$$

$R \perp p$, $R \perp q$

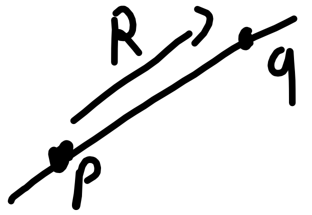


$$P\begin{pmatrix} x \\ y \\ z \end{pmatrix}_B, \quad b_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}_E, \quad b_2 = \ldots$$

$$x b_1 + y b_2 + z b_3$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{pmatrix}_E \begin{pmatrix} x \\ y \\ z \end{pmatrix}_B = \begin{pmatrix} \bar{x} \\ \bar{y} \\ \bar{z} \end{pmatrix}_E$$

Lines:



$$L = \{\ell \mid p + \alpha R\ ,\ p, R \in \mathbb{R}^3,\ \alpha \in \mathbb{R}\}$$

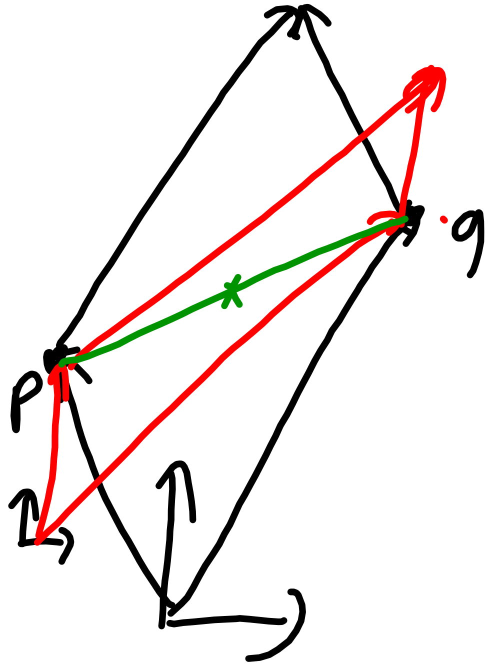
$$L = \{\ell \mid p + \alpha(q - p)$$

$$(1 - \alpha)p + \alpha q)$$

$$\alpha\ p + \beta q\ ,\quad \alpha + \beta = 1$$

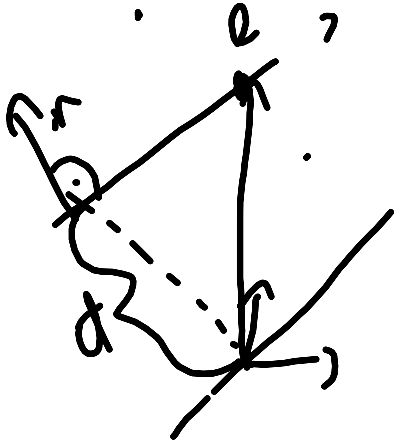
$p + q$

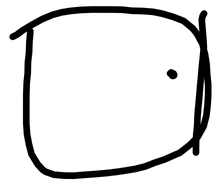
$\frac{1}{2}p + \frac{1}{2}q$



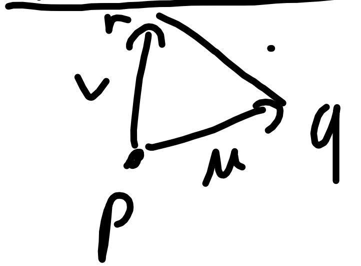
Implicit:

$\mathbb{R}^2: \quad L: \{ l \mid n^T l + d = 0 \}$





planes:



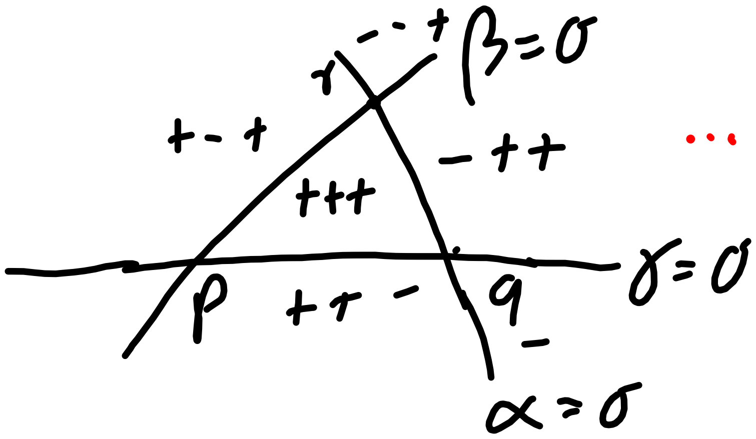
$$E = \{ l \mid l = p + \lambda \cdot u + \mu \cdot v \}$$

$$\{ l \mid l = p + \lambda(q-p) + \mu \cdot (r-p)$$

$$= (1 - \lambda - \mu) p + \lambda q + \mu r$$

$$t = \alpha \quad p + \beta q + \gamma r$$

$$\alpha + \beta + \gamma = 1$$



$\alpha p + \beta q + \gamma r$

$$E = \{ l \mid n^T l + d = 0 ,\ n, l \in \mathbb{R}^3$$

$$p \qquad L(p) + t \qquad p = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \qquad V = \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \alpha & 0 & 0 & t_x \\ 0 & \beta & 0 & t_y \\ 0 & 0 & \gamma & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha x + t_x \\ \beta y + t_y \\ \vdots \\ \vdots \end{pmatrix} \qquad \frac{1}{2} p + \frac{1}{2} q$$