2.6 Graphen

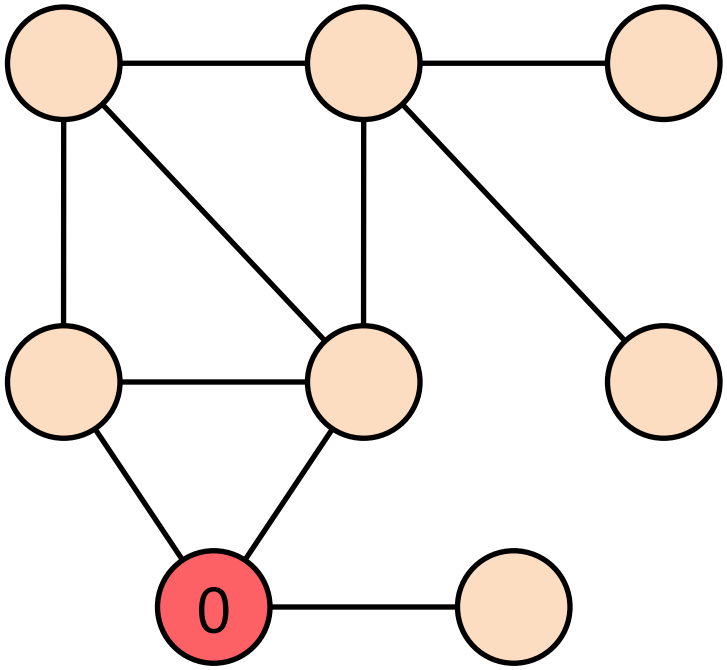
2.6.2 Ausspähen von Graphen

2.6.2.1 Breitensuche

2.6.2.2 Tiefensuche

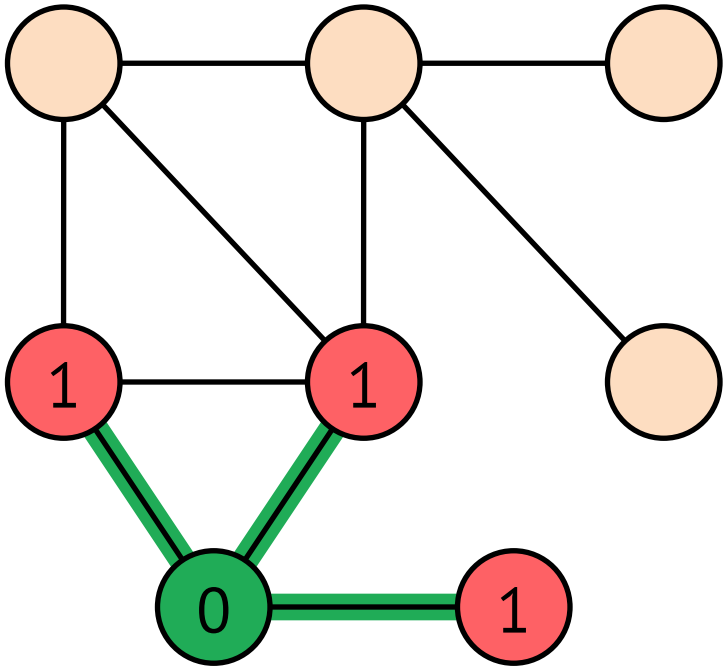
2.6.2.3 Topologisches Sortieren

Breitensuche



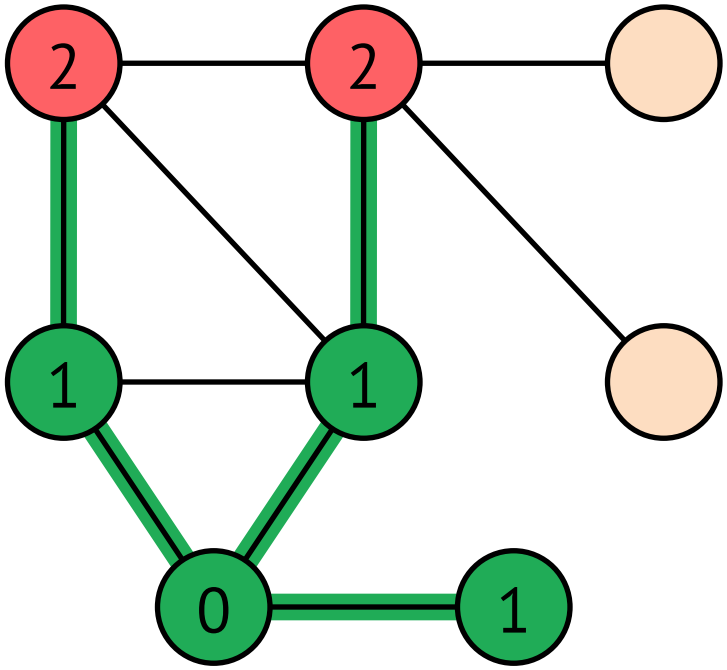
Quelle

Breitensuche



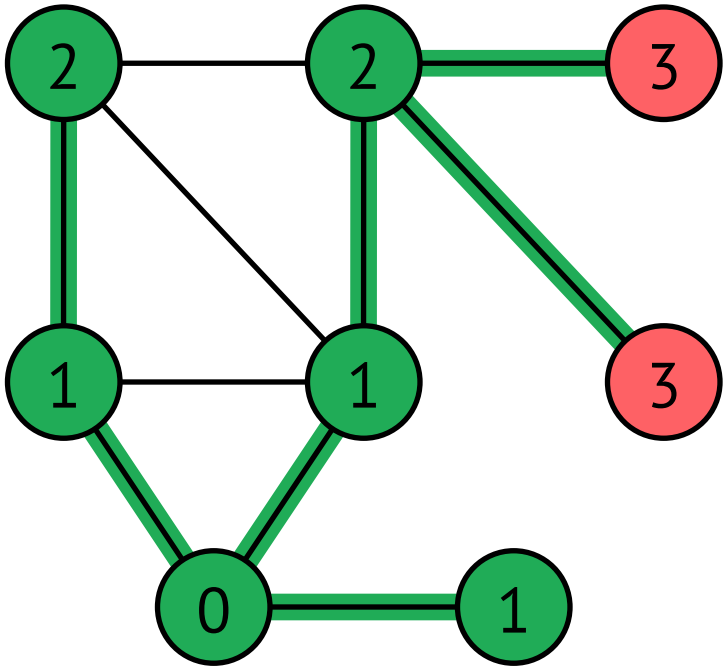
Quelle

Breitensuche



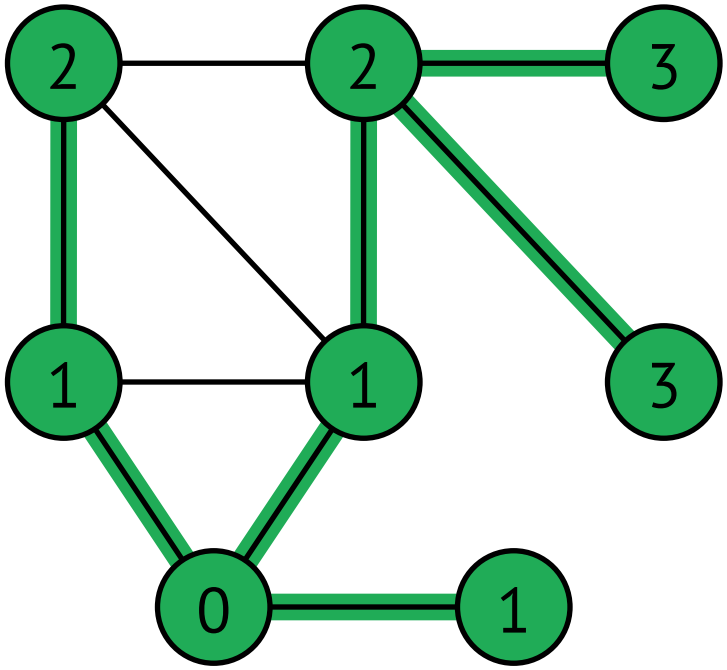
Quelle

Breitensuche



Quelle

Breitensuche



Quelle

Breitensuche

- Von einem gegebenen Startknoten s (Quelle, source) ausgehend, werden nacheinander alle von s erreichbaren Knoten besucht.
- Die Grenze zwischen besuchten und nicht besuchten Knoten wird gleichmäßig vorangetrieben (breadth-first)

Breitensuche

- Während der Breitensuche wird implizit oder explizit ein Breitensuchbaum aufgebaut.
- Die Tiefe eines Knotens u im Breitensuchbaum ist gleich der Länge des kürzesten Pfades von s zu u.
- Implementierung der Front durch eine Queue.

Analyse

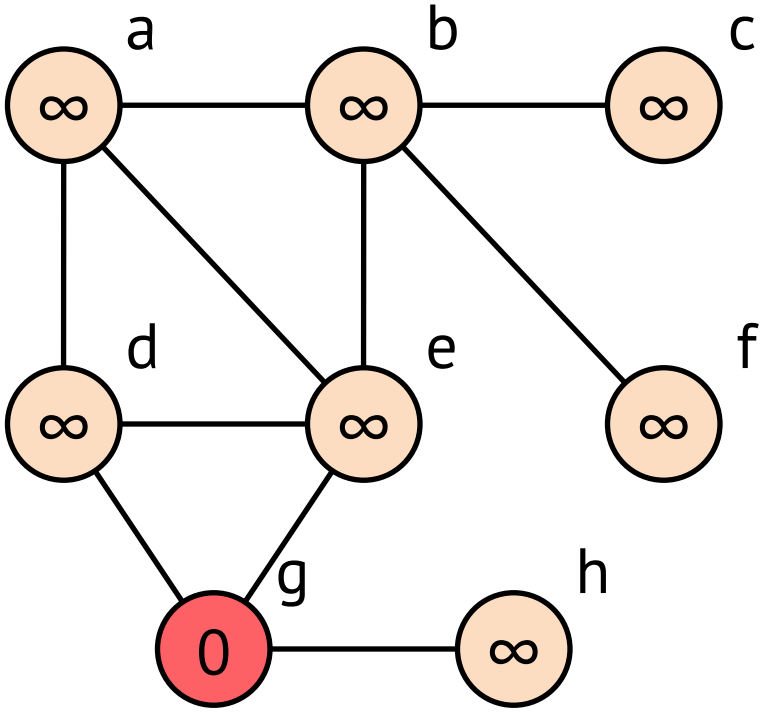
- BreadthFirstSearch(G)
  - for all v ∈ V do
    - depth[v] ← ∞
  - for all v ∈ V do
    - if depth[v] = ∞ then
      - VisitBreadthFirst(G, v)

Analyse

- VisitBreadthFirst(G, s)

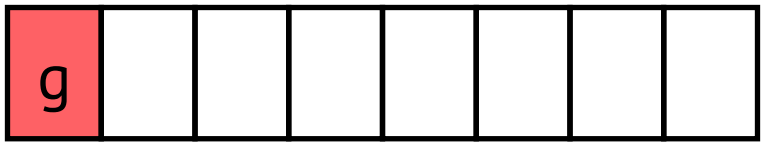
```
depth[ s ] ← 0
Q ← {}
enqueue( Q, s )
while Q not empty do
    u ← dequeue( Q )
    for all v ∈ Adj[ u ] do
        if depth[ v ] = ∞ then
            depth[ v ] ← depth[ u ] + 1
            enqueue( Q, v )
```

Breitensuche

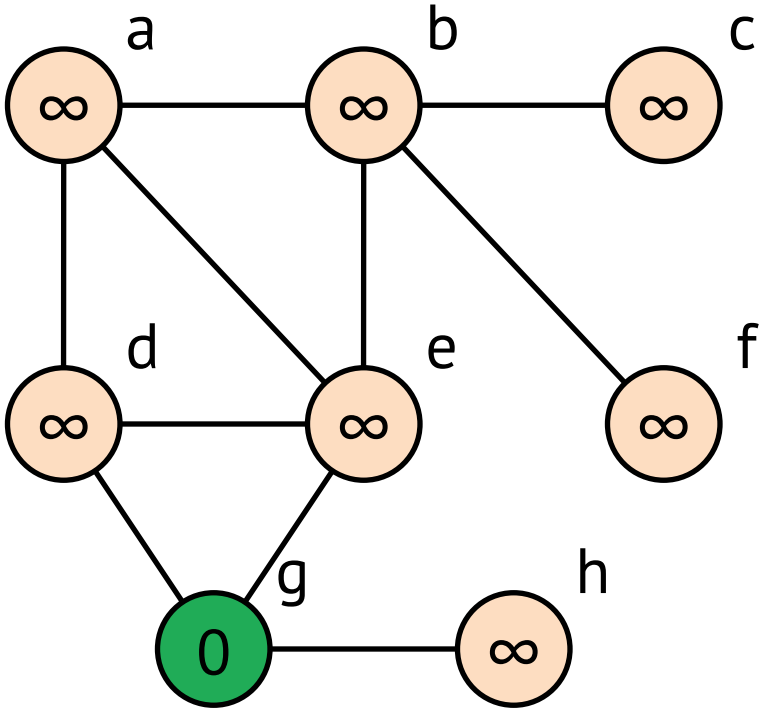


Quelle

Q

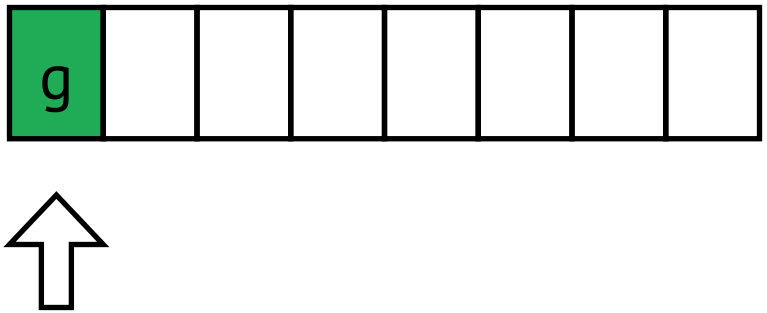


Breitensuche

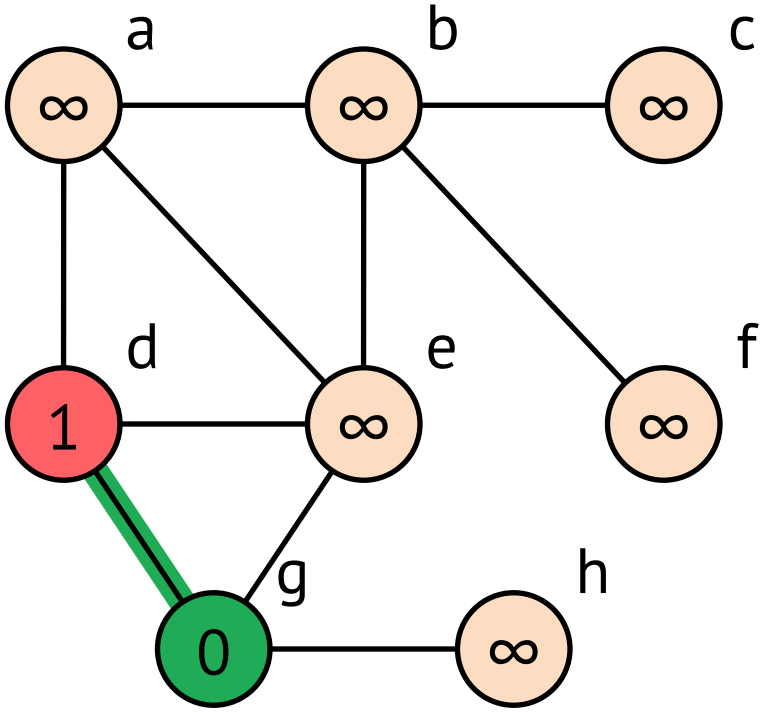


Quelle

Q

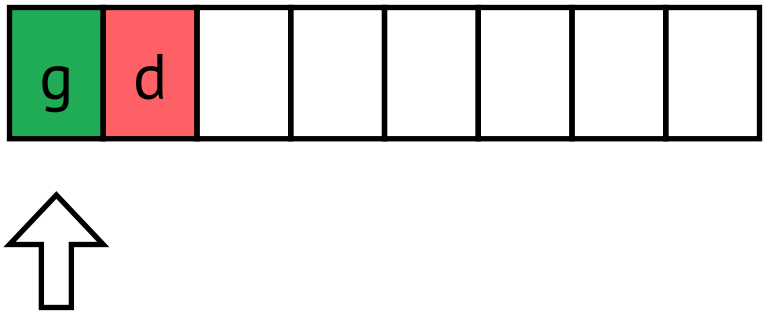


Breitensuche

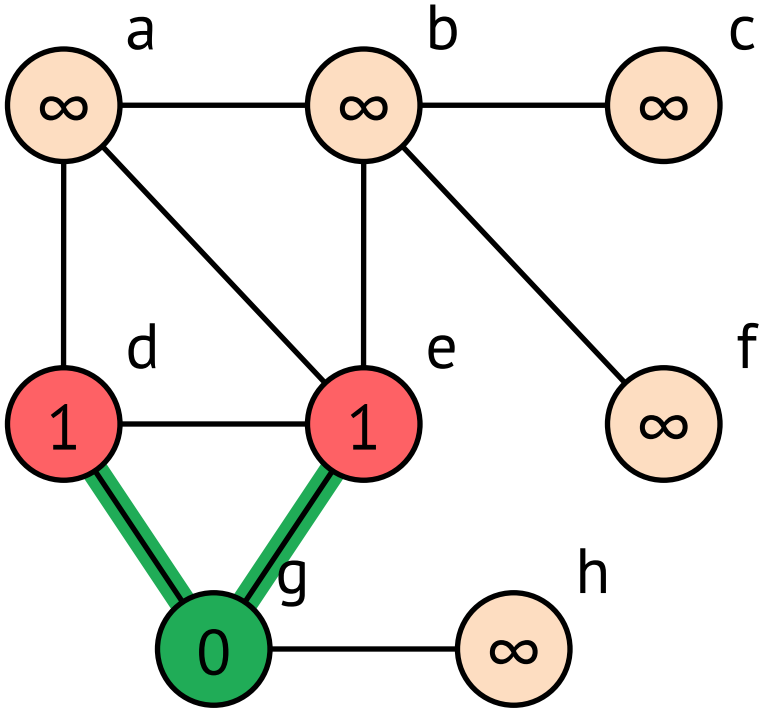


Quelle

Q

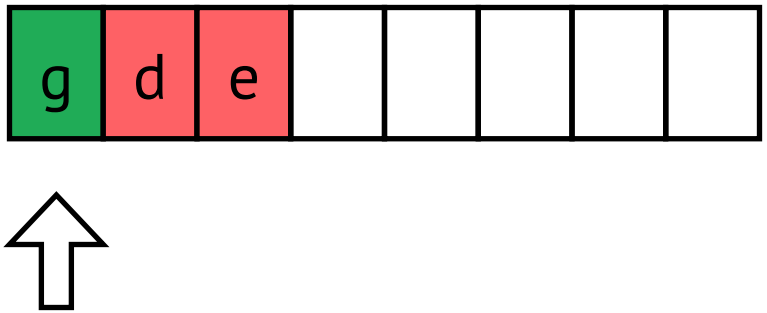


Breitensuche

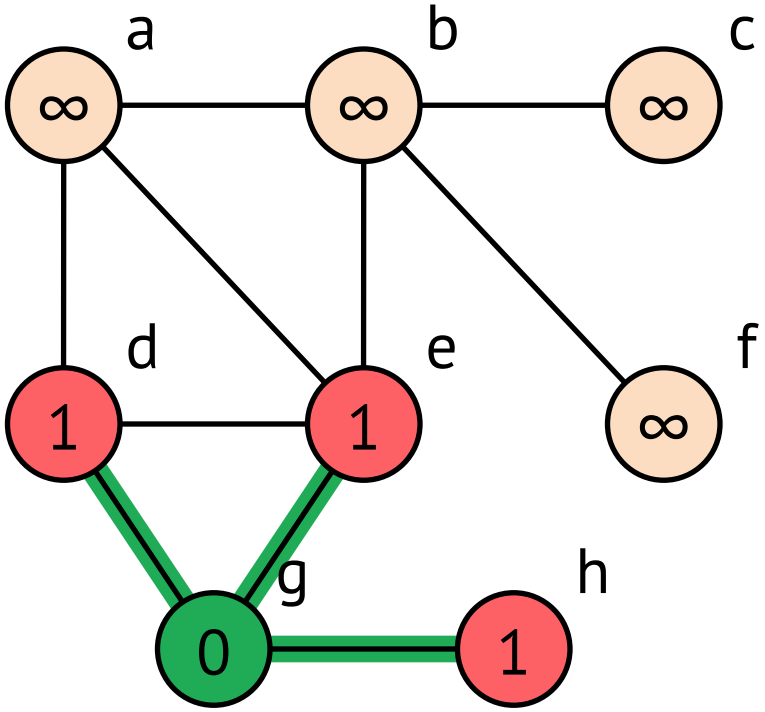


Quelle

Q

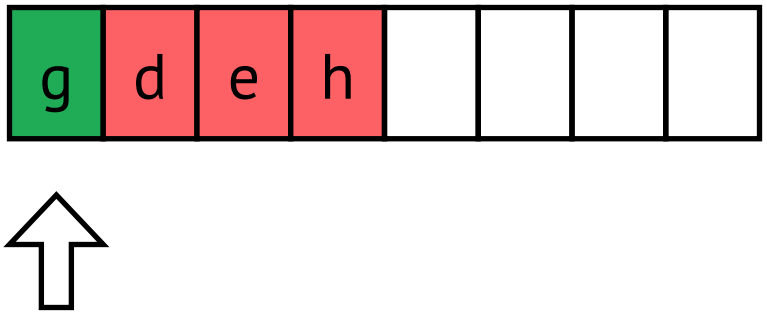


Breitensuche

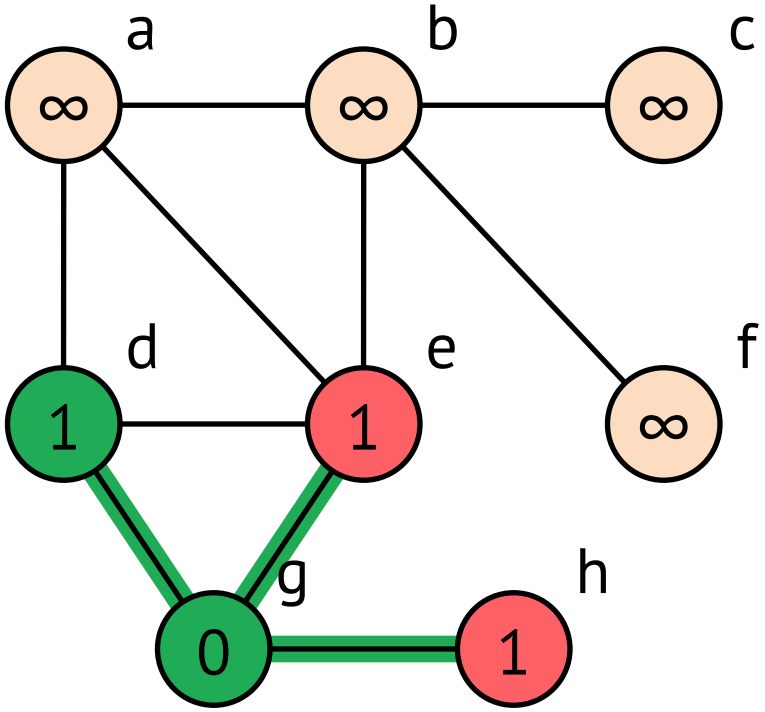


Quelle

Q

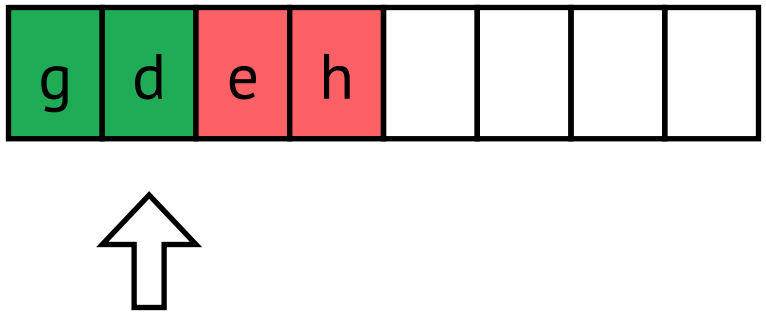


Breitensuche

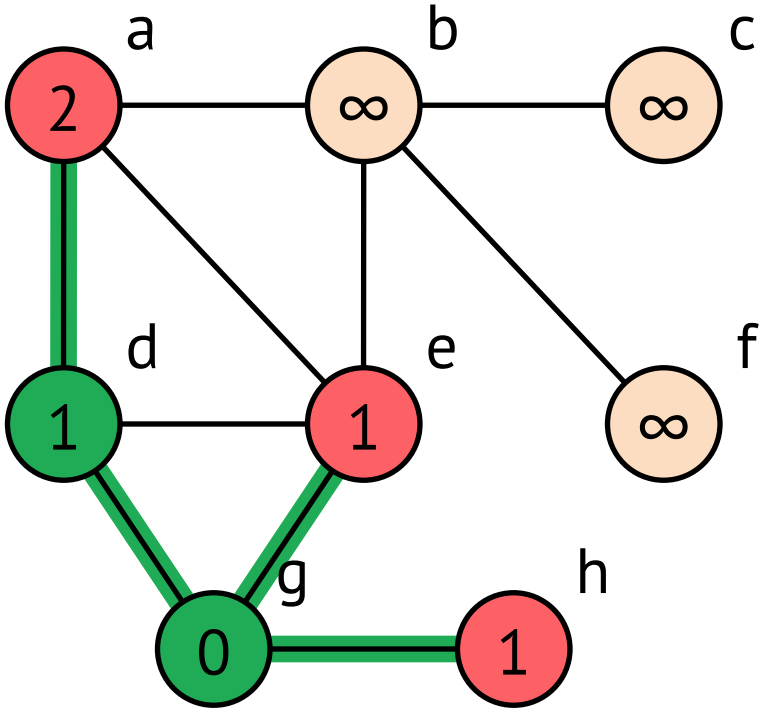


Quelle

Q

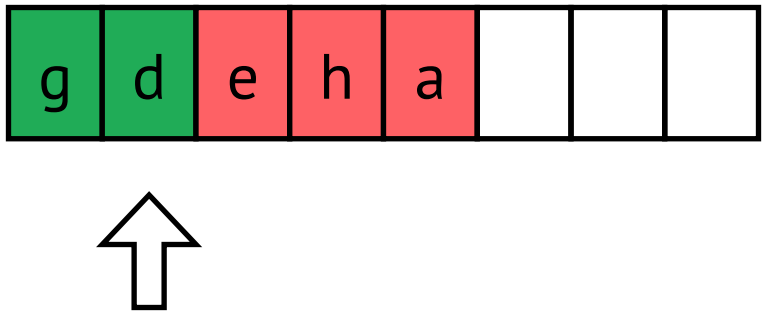


Breitensuche

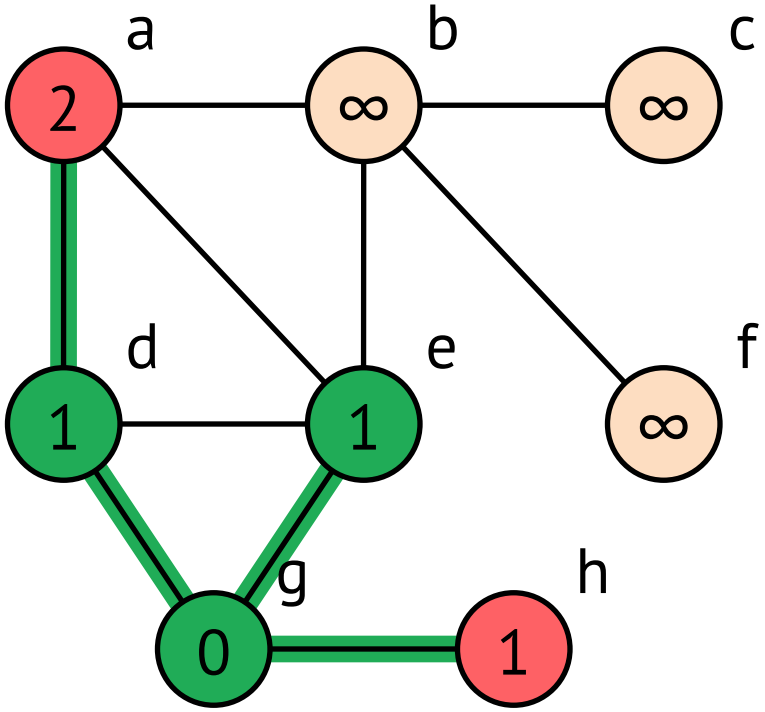


Quelle

Q

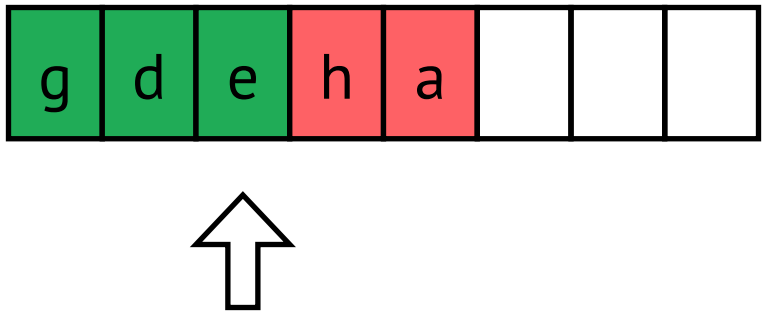


Breitensuche

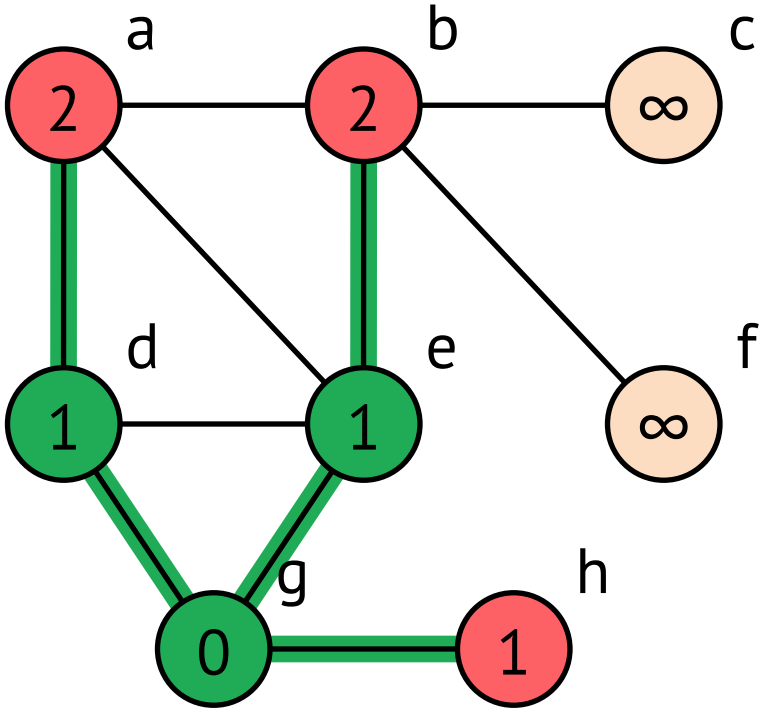


Quelle

Q

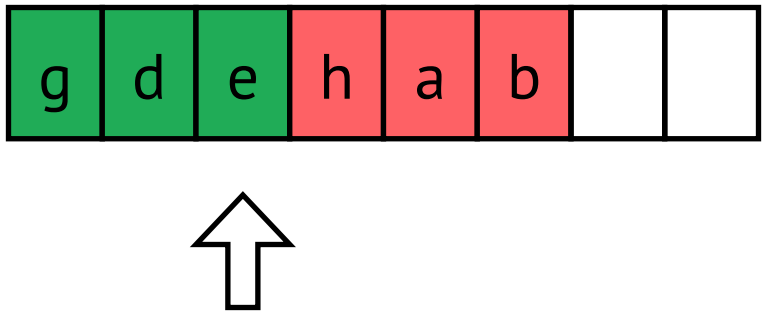


Breitensuche

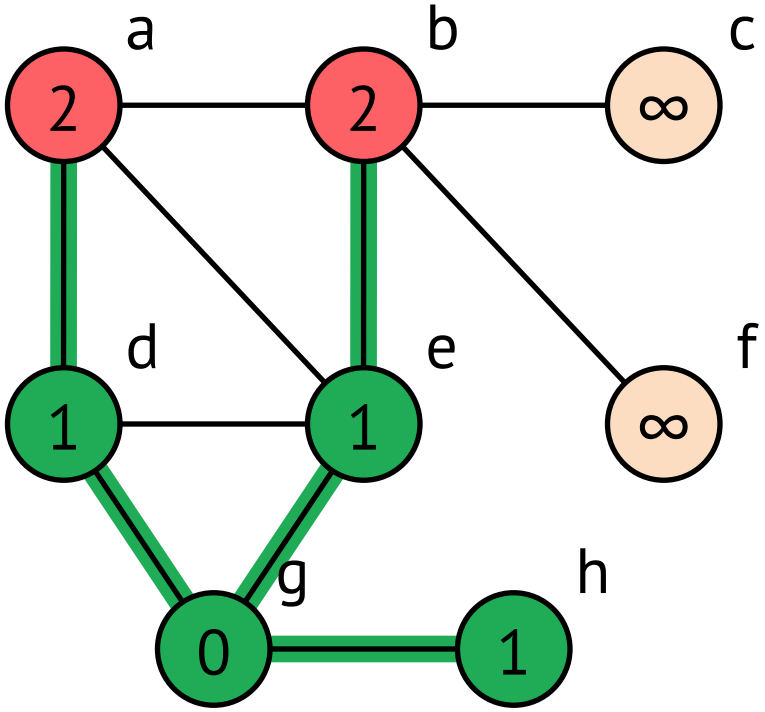


Quelle

Q

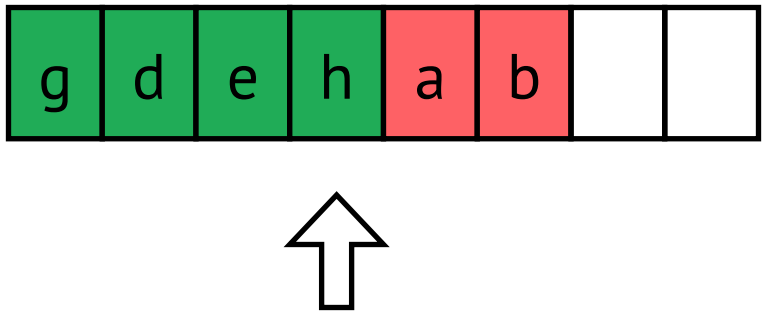


Breitensuche

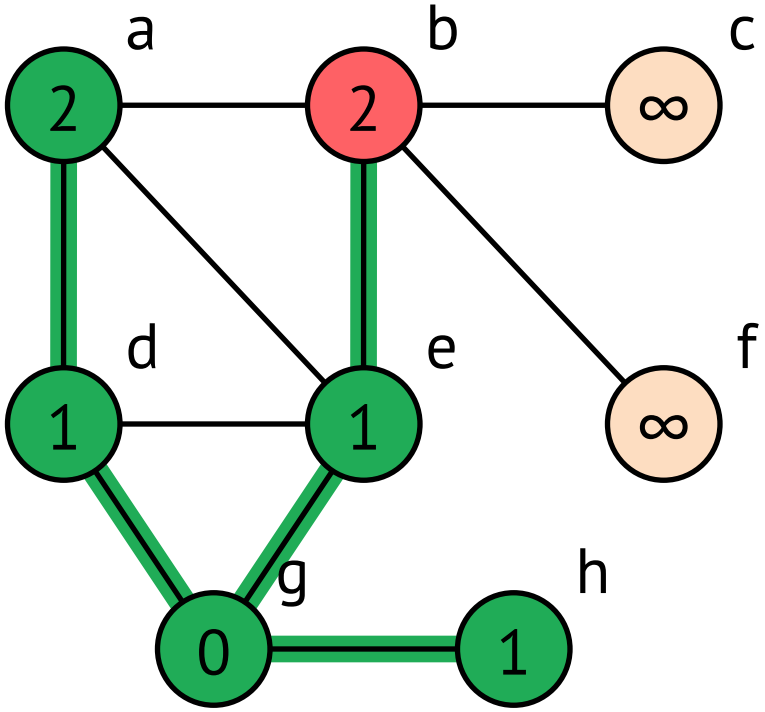


Quelle

Q

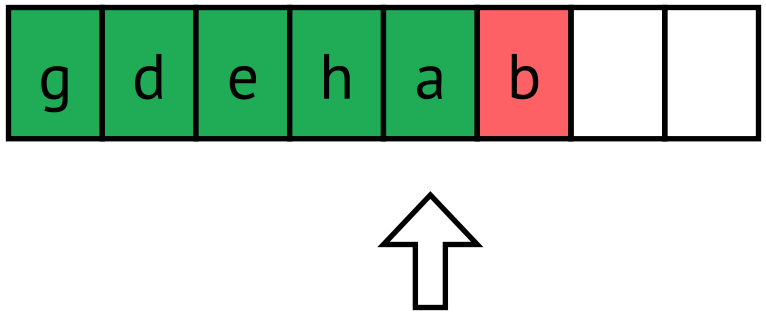


Breitensuche

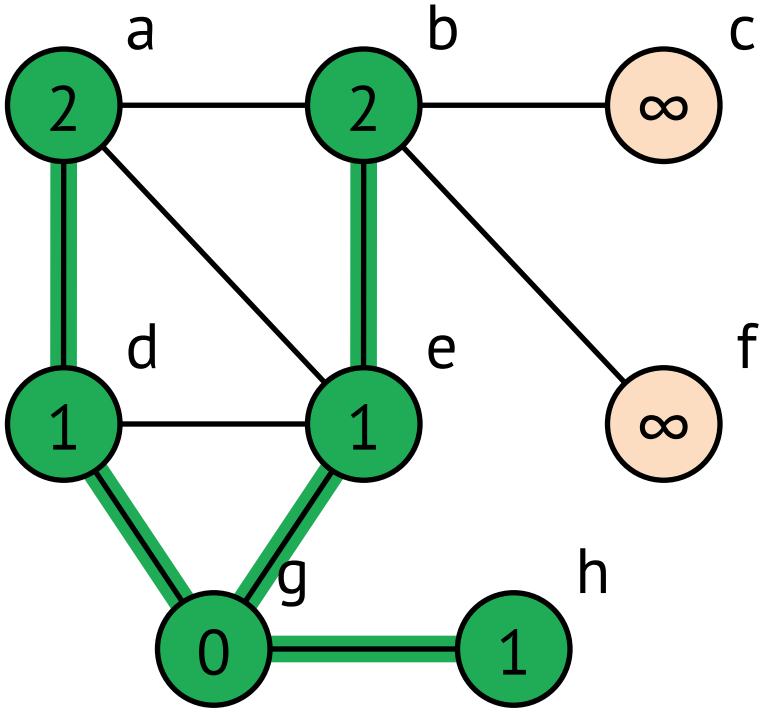


Quelle

Q

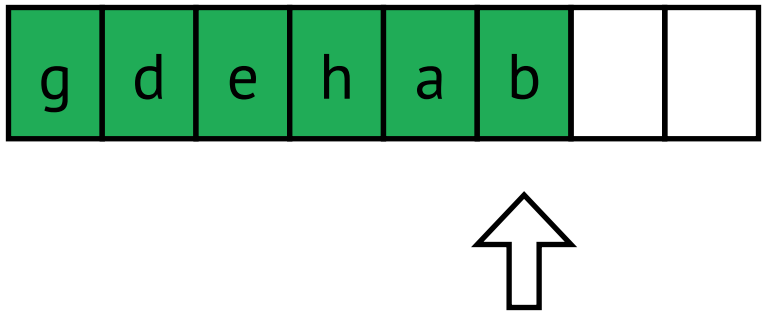


Breitensuche

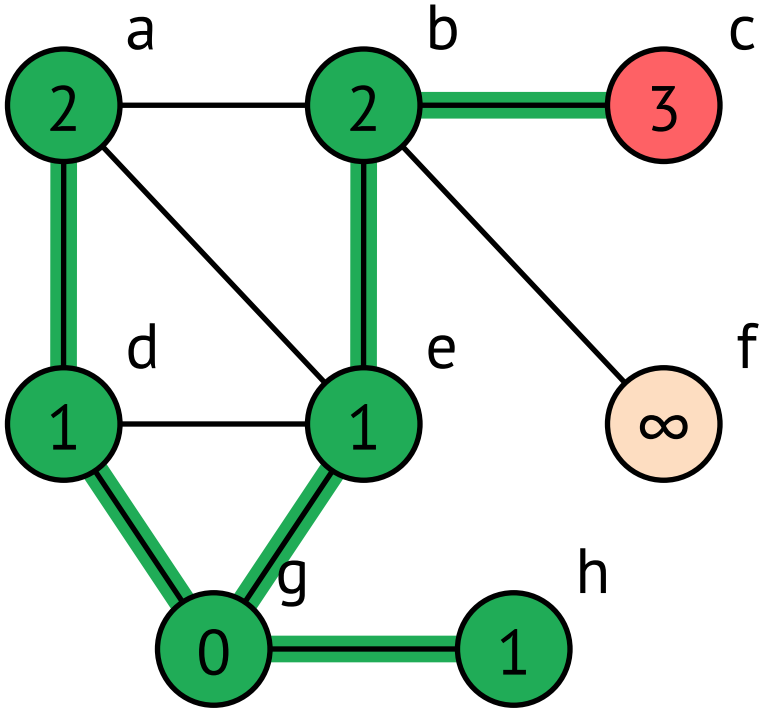


Quelle

Q

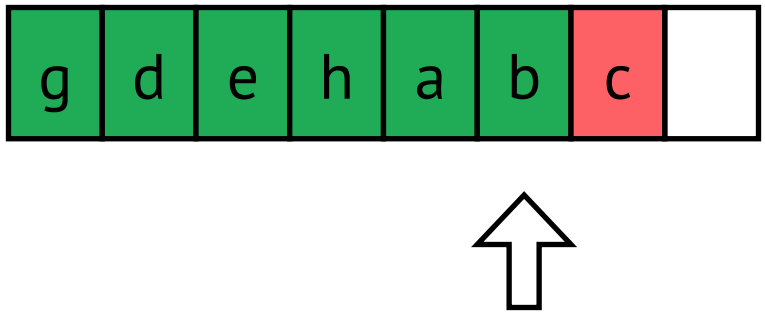


Breitensuche

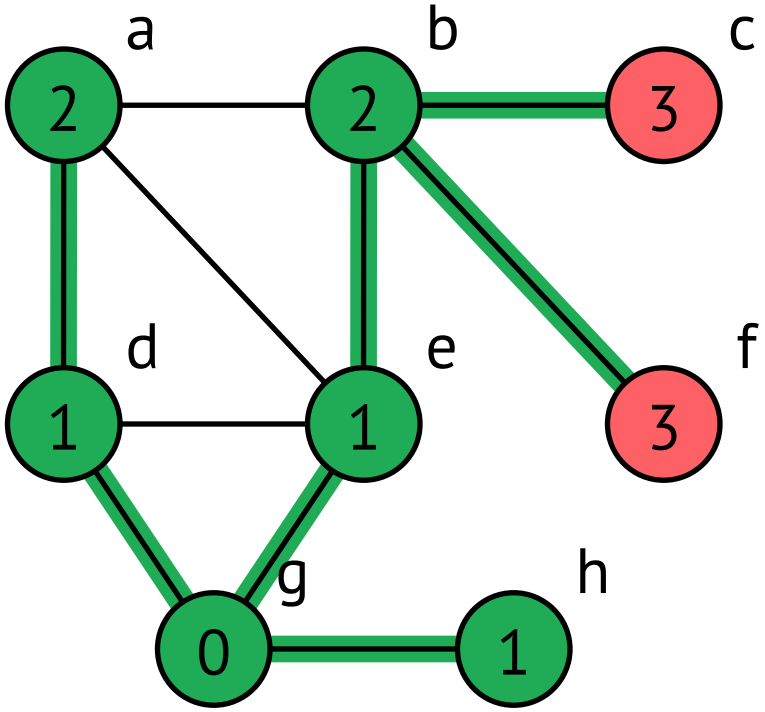


Quelle

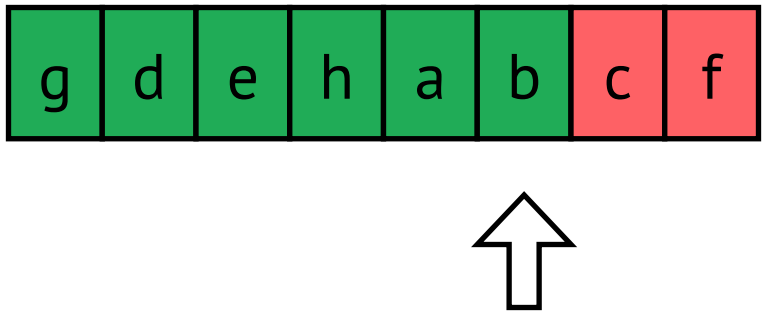
Q



Breitensuche

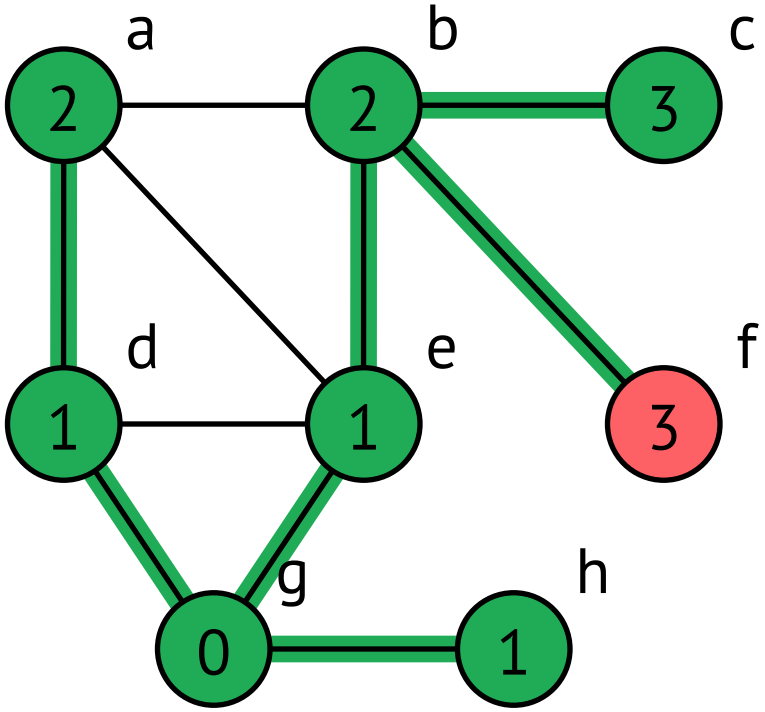


Q

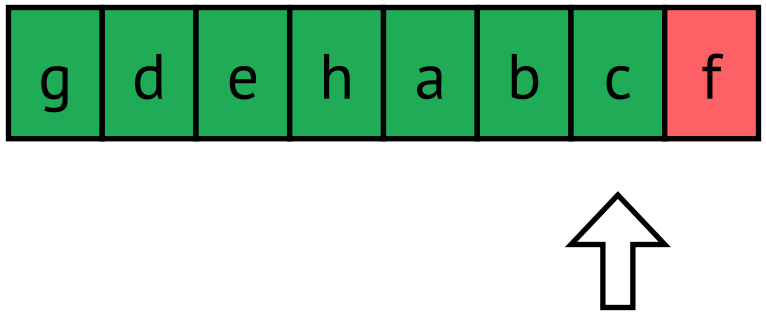


Quelle

Breitensuche

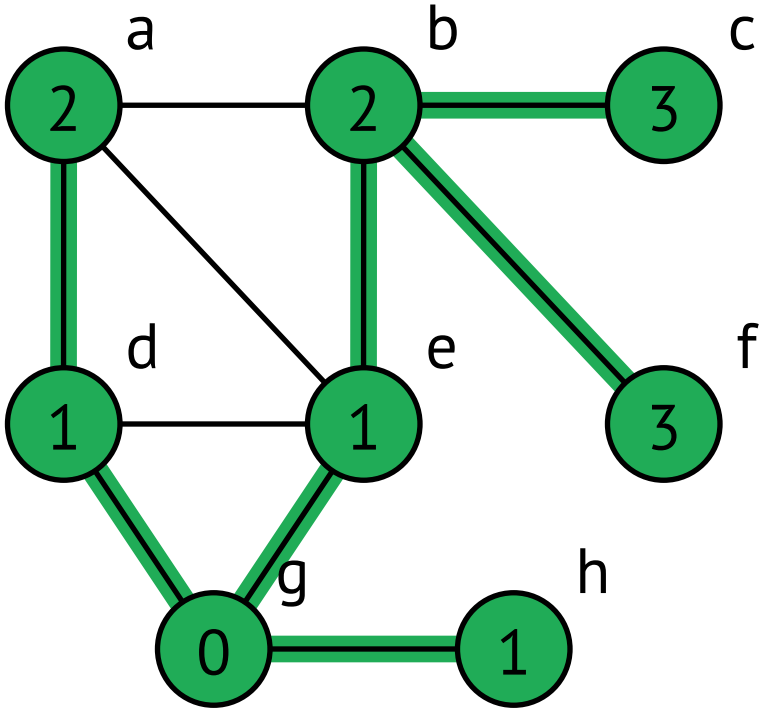


Q

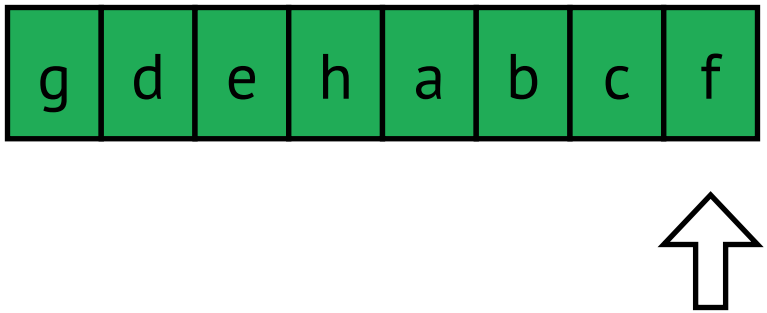


Quelle

Breitensuche

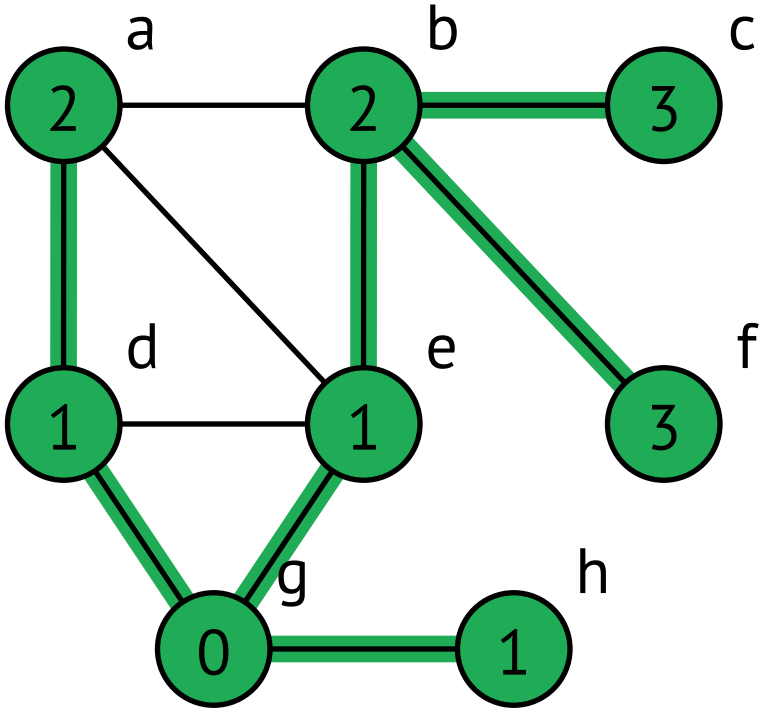


Q

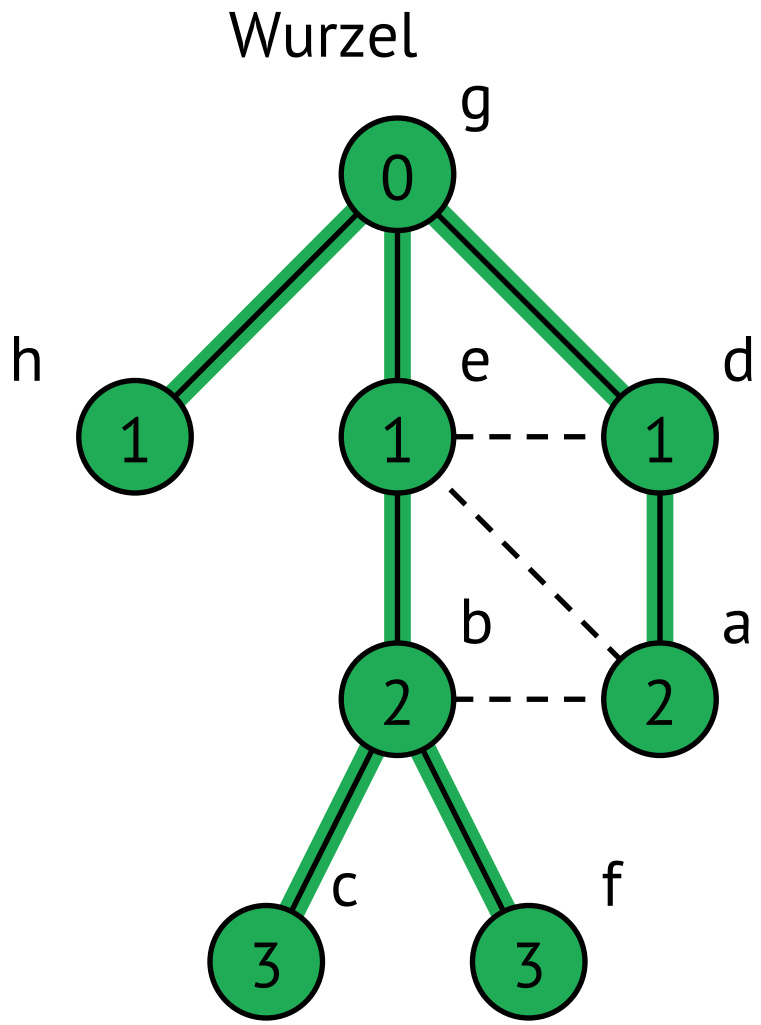


Quelle

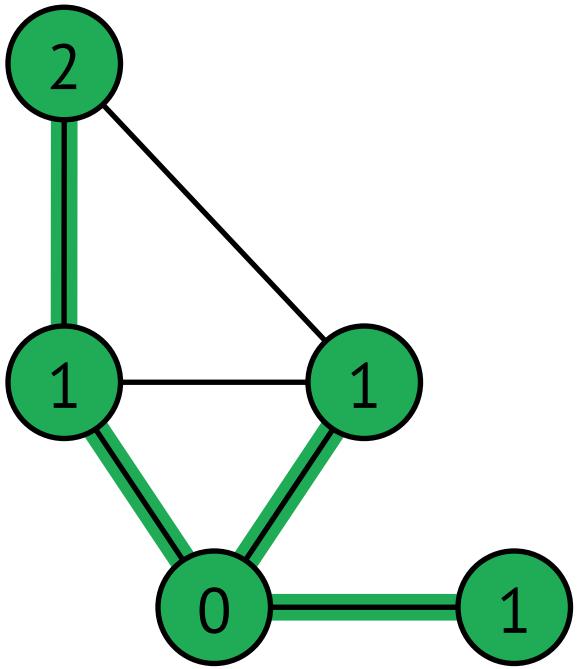
Breitensuche

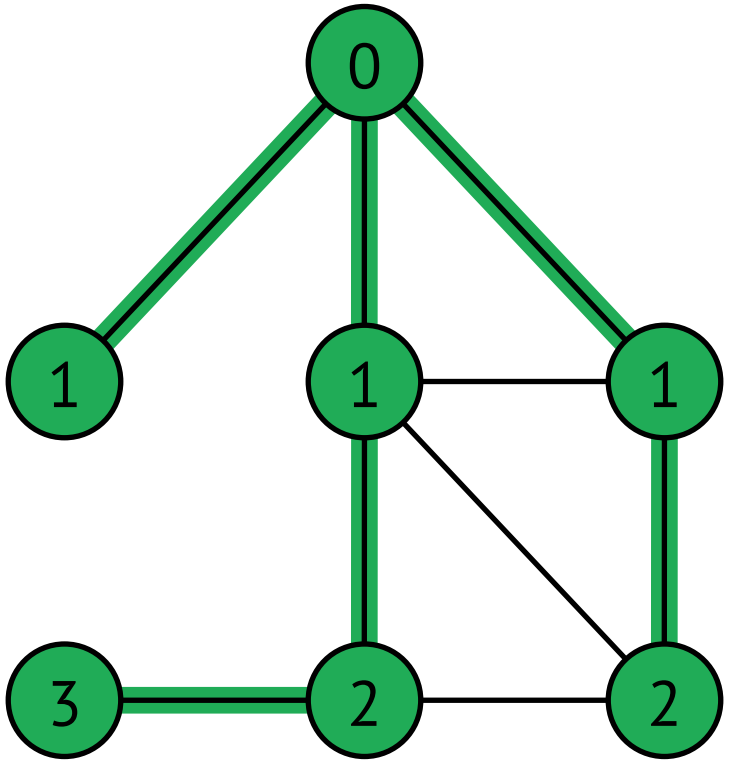


Quelle



Breitensuchwald





Analyse

- BreadthFirstSearch(G)

```
for all v ∈ V do
    depth[ v ] ← ∞
for all v ∈ V do
    if depth[ v ] = ∞ then
        VisitBreadthFirst( G, v )
```

- Aufwand: O(V)

Analyse

- VisitBreadthFirst(G, s)

```
depth[ s ] ← 0
Q ← {}
enqueue( Q, s )
while Q not empty do
    u ← dequeue( Q )
    for all v ∈ Adj[ u ] do
        if depth[ v ] = ∞ then
            depth[ v ] ← depth[ u ] + 1
            enqueue( Q, v )
```

Wie oft werden diese innersten Anweisungen ausgeführt?

Analyse

- Jeder Knoten wird genau einmal in die Queue geschoben.
- Jeder Knoten wird genau einmal aus der Queue genommen.
- Adj[u] wird genau dann abgearbeitet, wenn u aus der Queue genommen wird.
- Der Aufwand zur Abarbeitung der Adjazenzlisten ist also $\sum_{u \in V} |Adj[u]| = O(E)$

Analyse

- Gesamtaufwand der Breitensuche

$$O(V+E)$$

Eigenschaften

- Die Abstände zwischen allen Knotenpaaren eines Graphen lassen sich in Zeit $O(V \times (V+E))$ berechnen.
- Die Zahl der Zusammenhangs-komponenten eines ungerichteten Graphen läßt sich in $O(V+E)$ berechnen.
- Jeder ungerichtete Graph kann in $O(V+E)$ auf Zusammenhang getestet werden.

Breitensuche

- COMPONENTS(G)

```
for all v ∈ V do
    comp[ v ] ← 0
components ← 0
for all v ∈ V do
    if comp[ v ] = 0 then
        components ← components + 1
        VISITBREADTHFIRST'( G, v )
return components
```

Breitensuche

- VisitBreadthFirst'(G, s)

```
comp[ s ] ← components
Q ← {}
enqueue( Q, s )
while Q not empty do
    u ← dequeue( Q )
    for all v ∈ Adj[ u ] do
        if comp[ v ] = 0 then
            comp[ v ] ← components
            enqueue( Q, v )
```

Eigenschaften

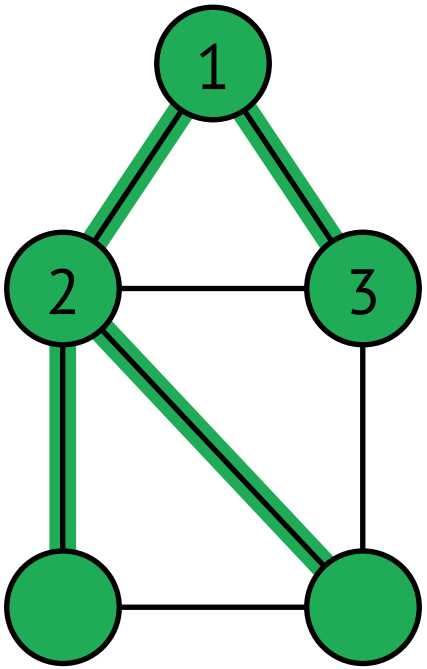
- Ein ungerichteter Graph lässt sich in $O(V)$ auf Kreisfreiheit testen.
- Erinnerung: Ein ungerichteter Graph $G=(V,E)$ mit k Zusammenhangskomponenten enthält einen Kreis, genau dann, wenn $E>V-k$
- Ist $E>V$ (nachzählen und abbrechen sobald Zähler größer V), so enthält G einen Kreis.
- Ist $E\leqslant V$ so zählt man die Komponenten wie oben beschrieben und zwar nur noch mit Aufwand $O(V+E)=O(V)$.

Eigenschaften

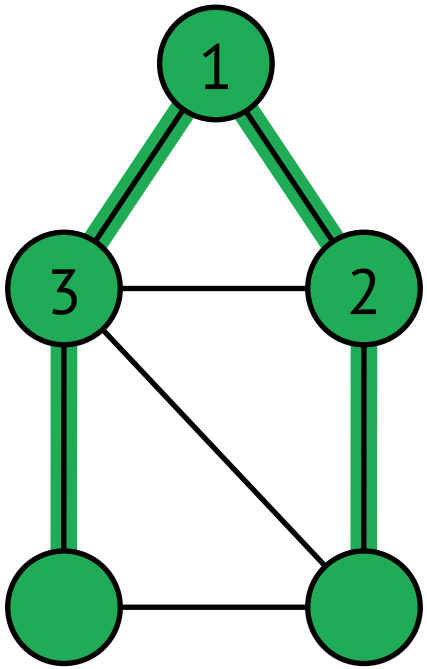
- Breitensuche berechnet für jeden Knoten u die Länge des kürzesten Pfades von der Quelle s zu u.
- Der Breitensuchbaum ist nicht eindeutig, sondern hängt von der Reihenfolge der Knoten in den Adjazenzlisten ab.

Eigenschaften

Quelle



Quelle



2.6.2 Ausspähen von Graphen

2.6.2.1 Breitensuche

2.6.2.2 Tiefensuche

2.6.2.3 Topologisches Sortieren

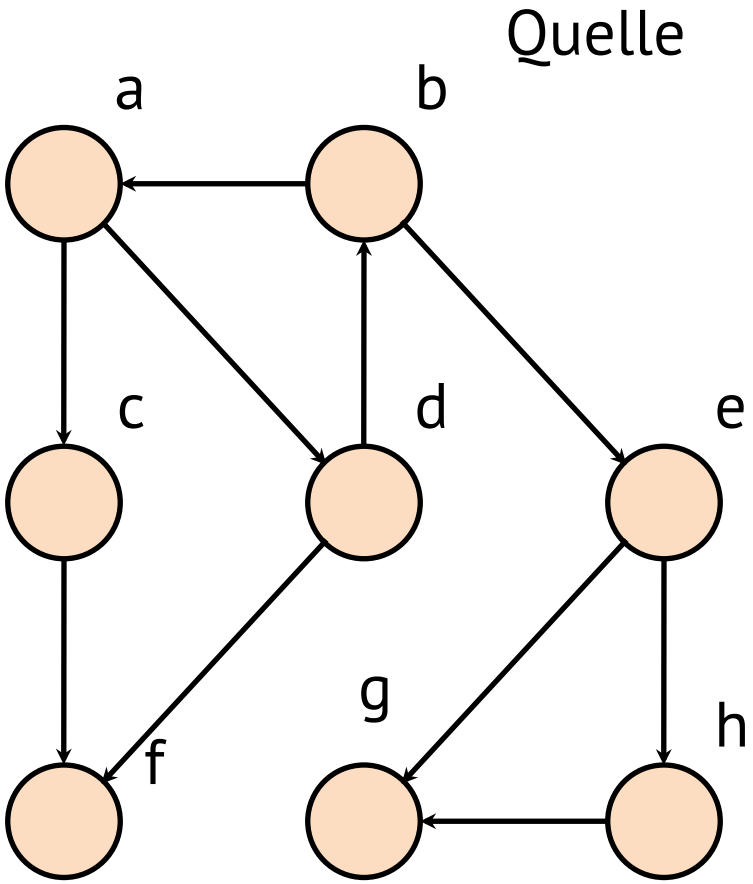
Tiefensuche

- Von einem gegebenen Startknoten s (Quelle, source) ausgehend, werden nacheinander alle von s erreichbaren Knoten besucht.
- Vom zuletzt besuchten Knoten werden zunächst die folgenden Knoten besucht (“depth-first”), Backtracking, falls alle Nachbarn besucht.

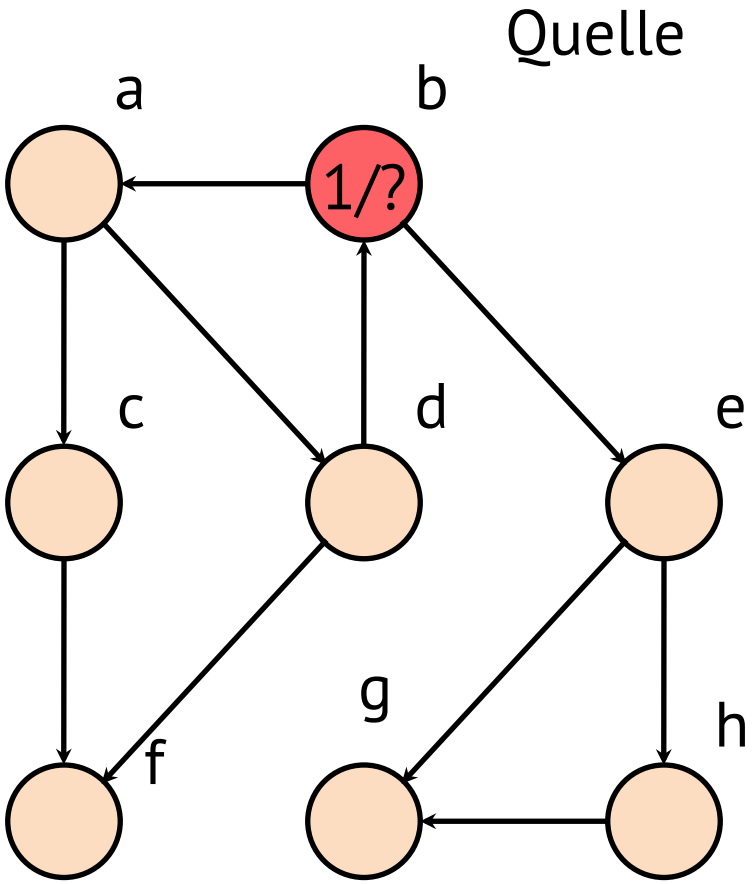
Tiefensuche

- In jedem Knoten werden die Eintritts- und Austrittszeitpunke gespeichert. Dies entspricht einer Präfix- bzw. Postfixsortierung auf dem entstehenden Tiefensuchwald.

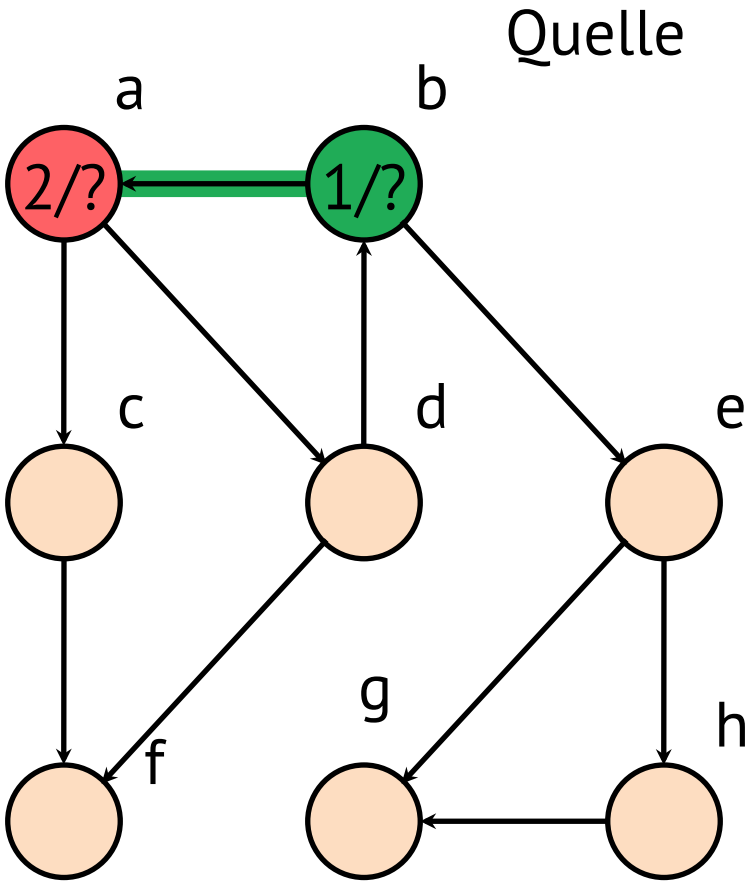
Beispiel



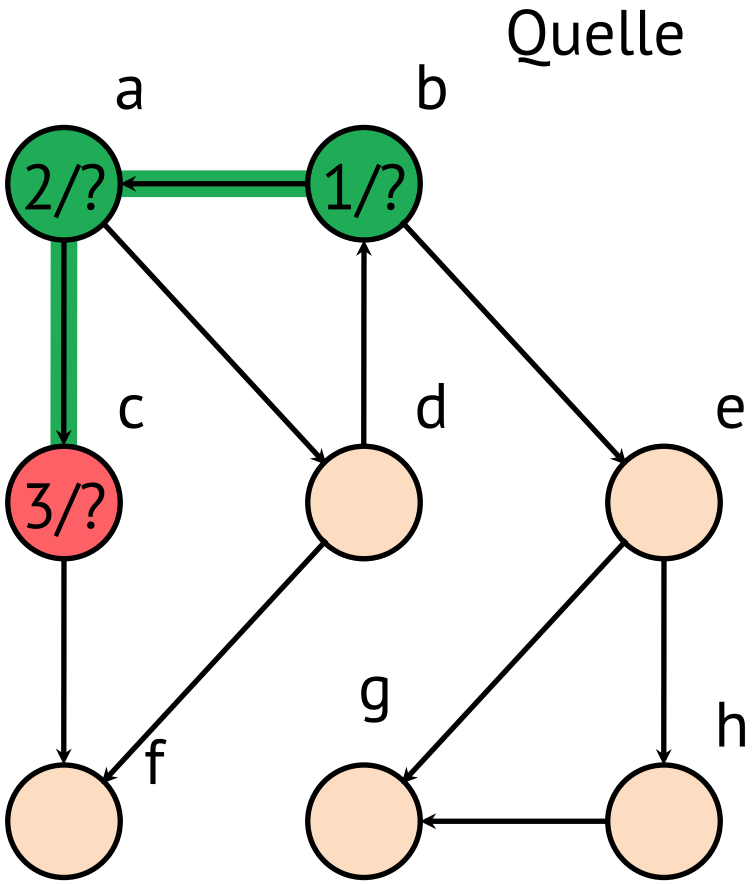
Beispiel



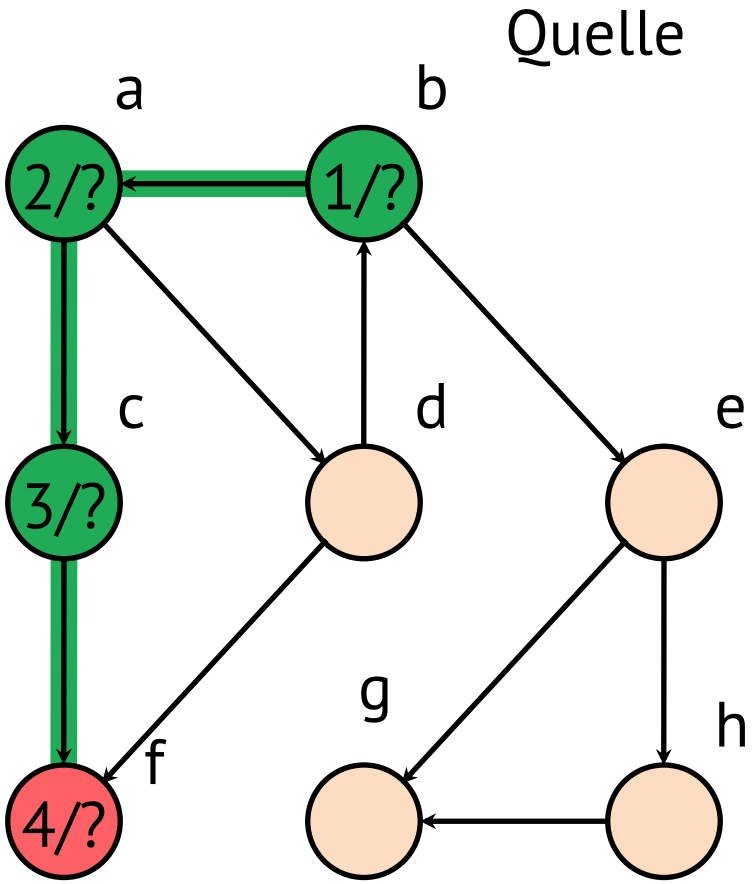
Beispiel



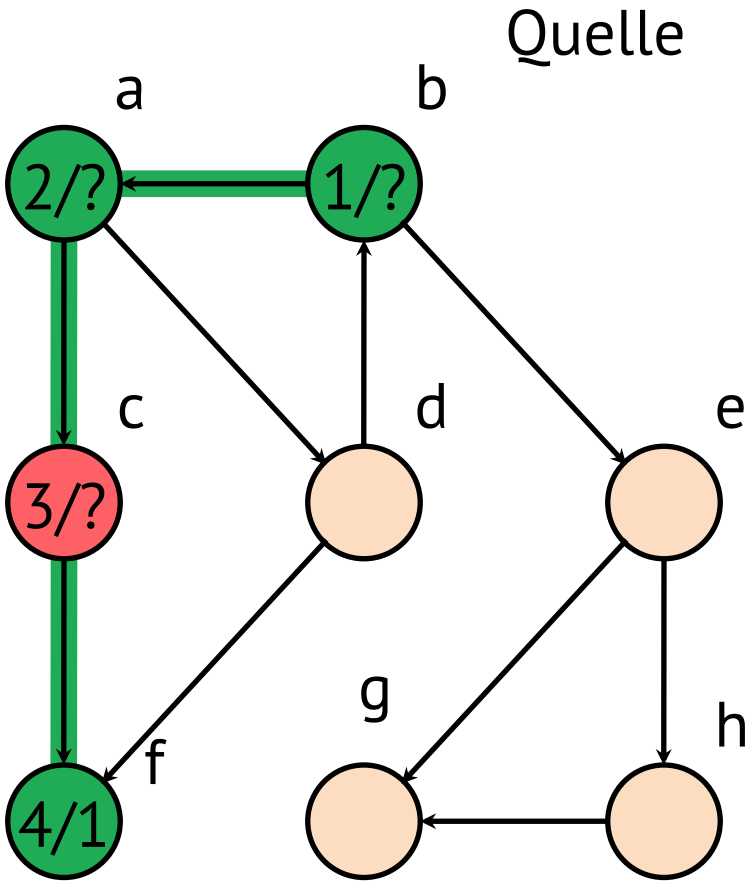
Beispiel



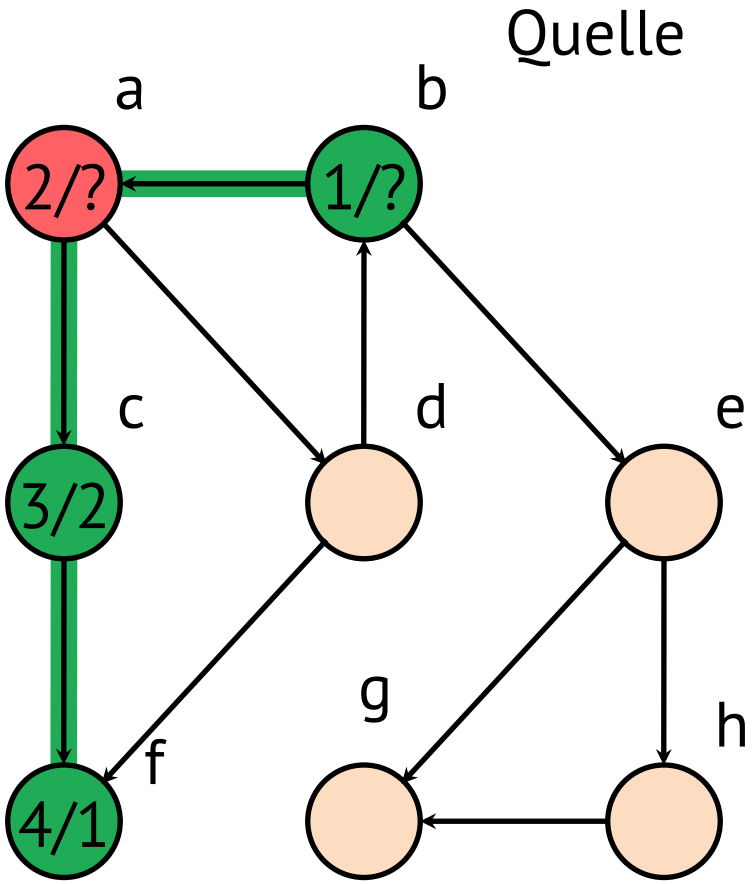
Beispiel



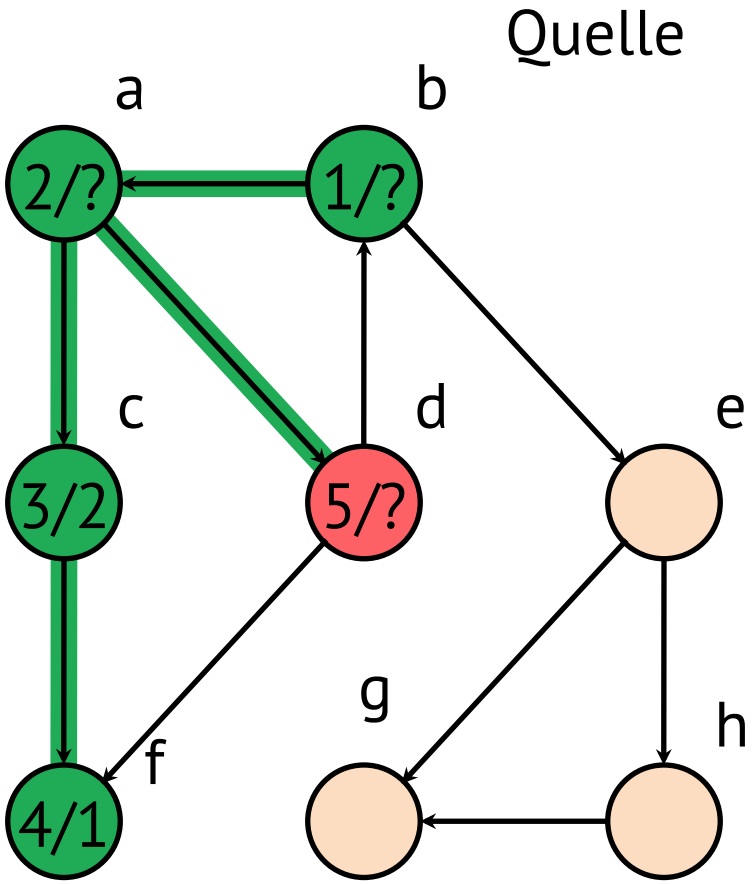
Beispiel



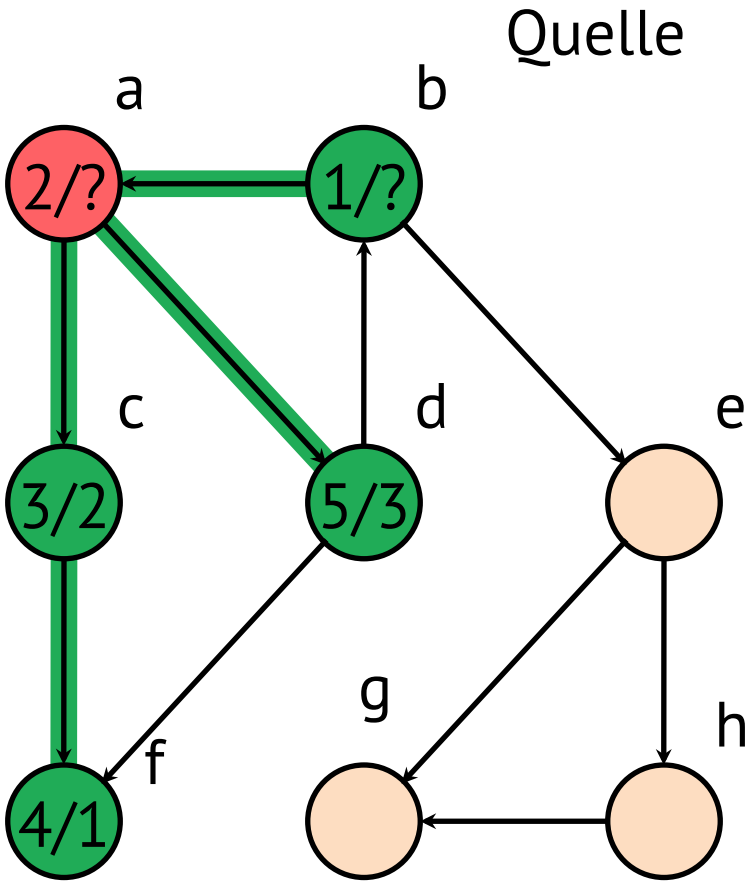
Beispiel



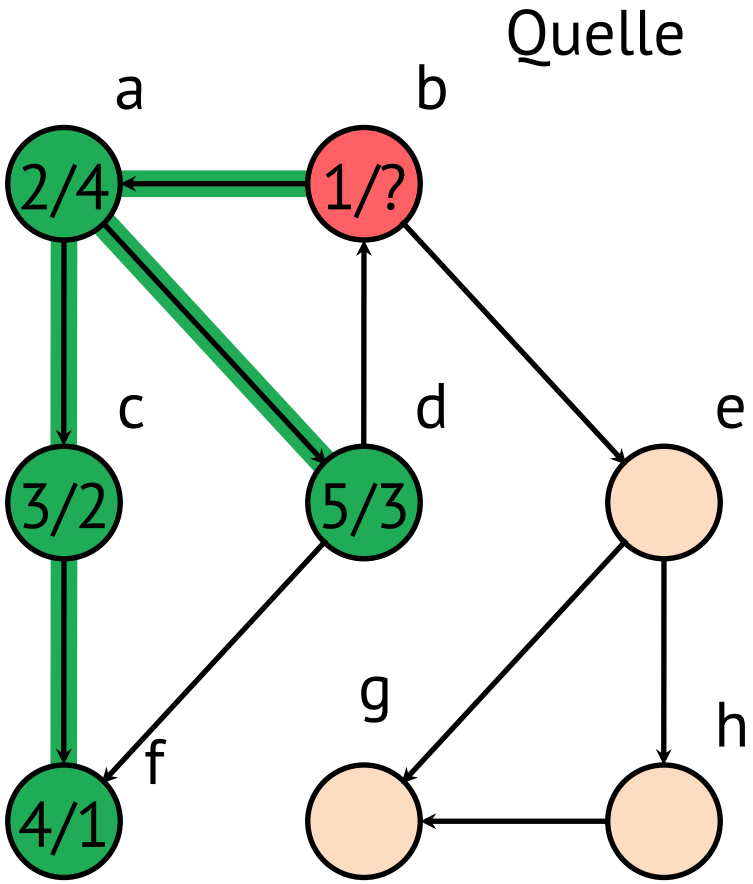
Beispiel



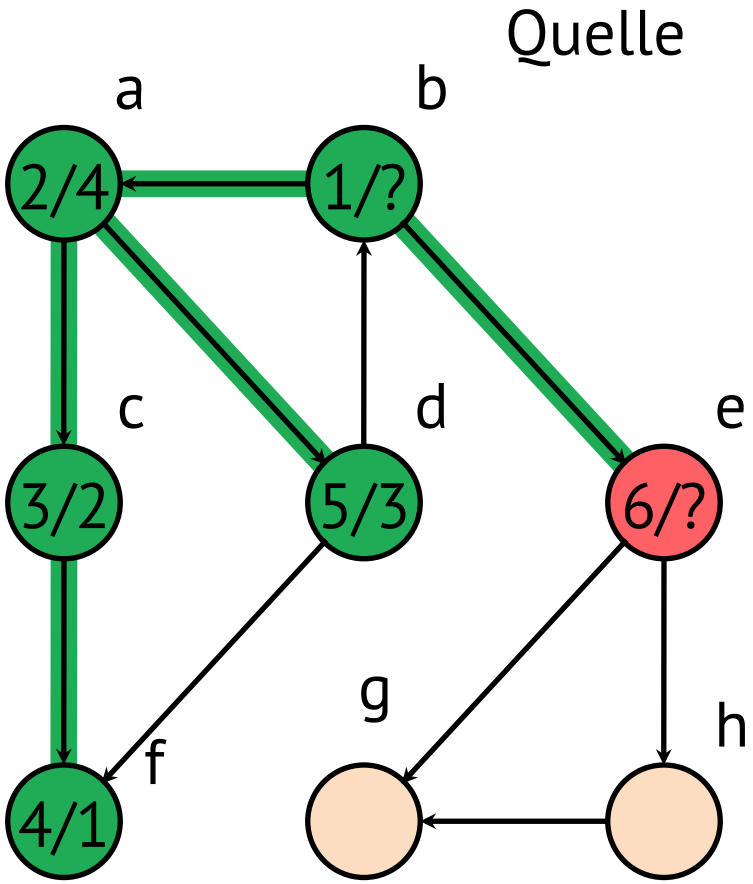
Beispiel



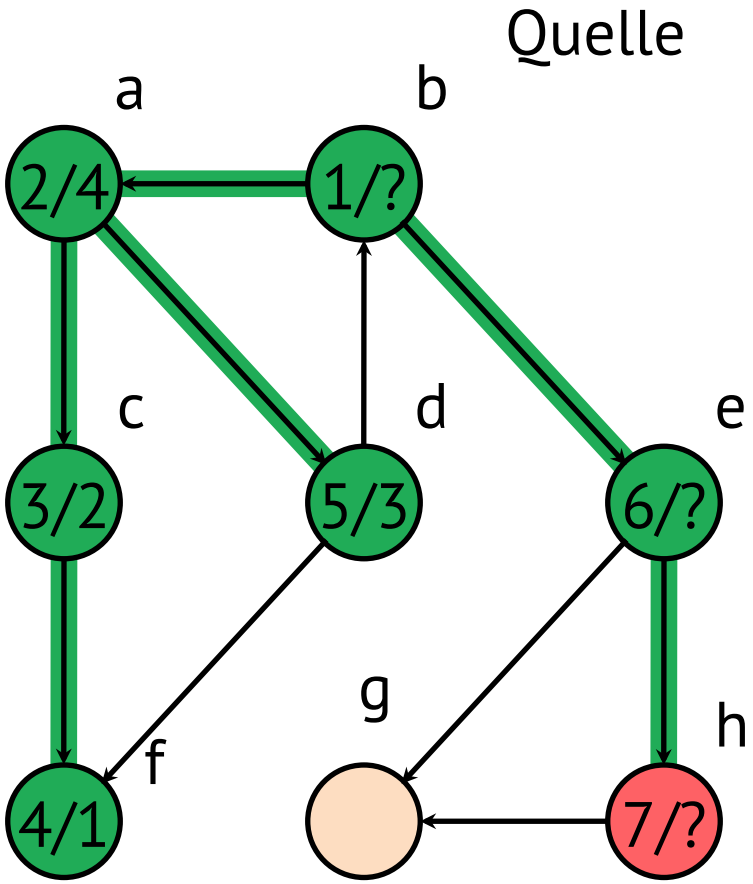
Beispiel



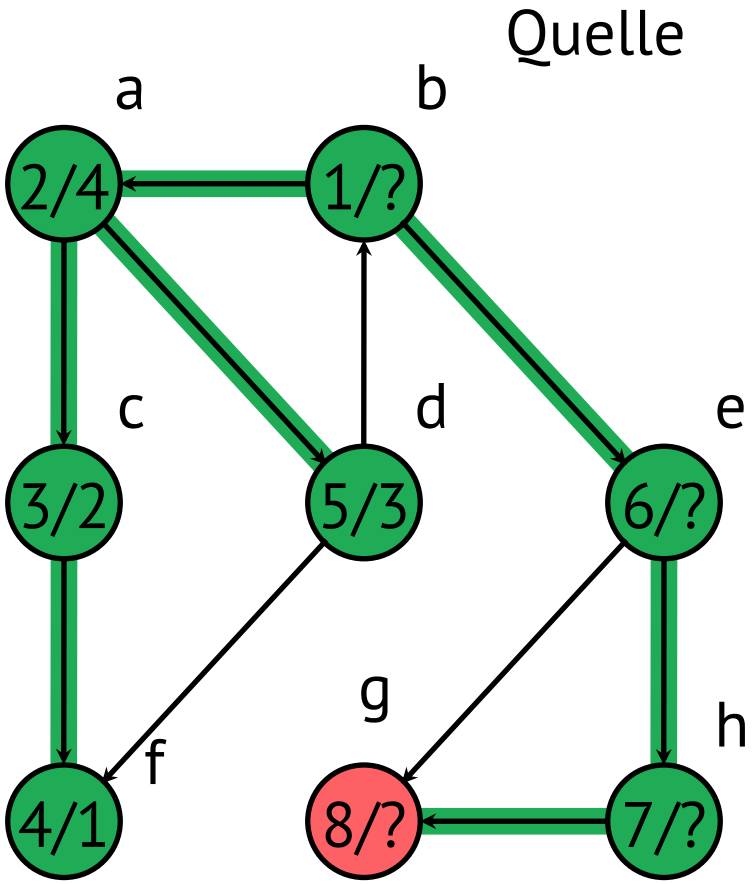
Beispiel



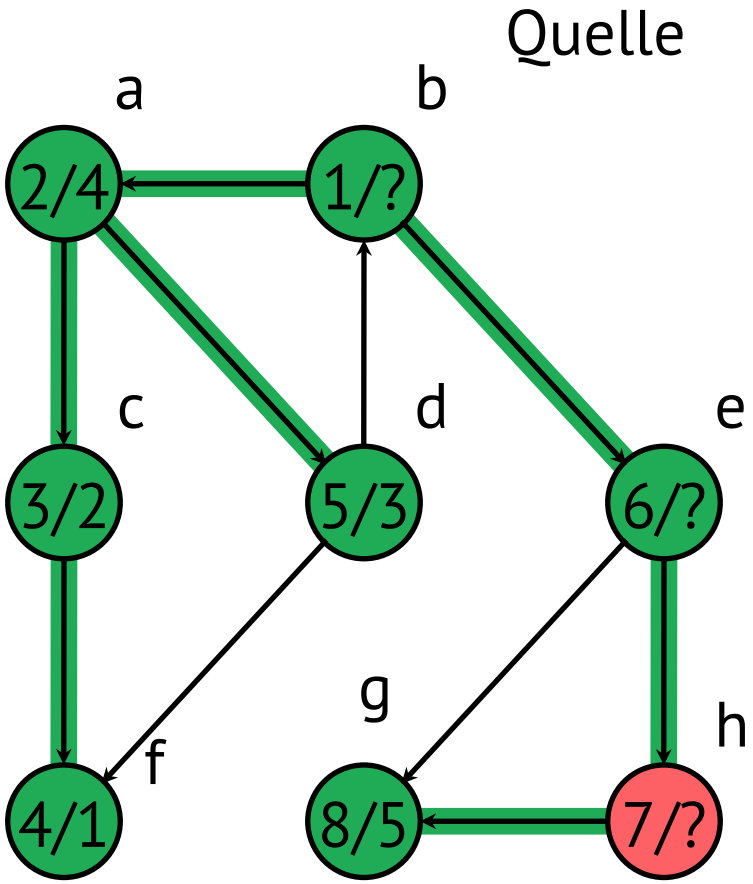
Beispiel



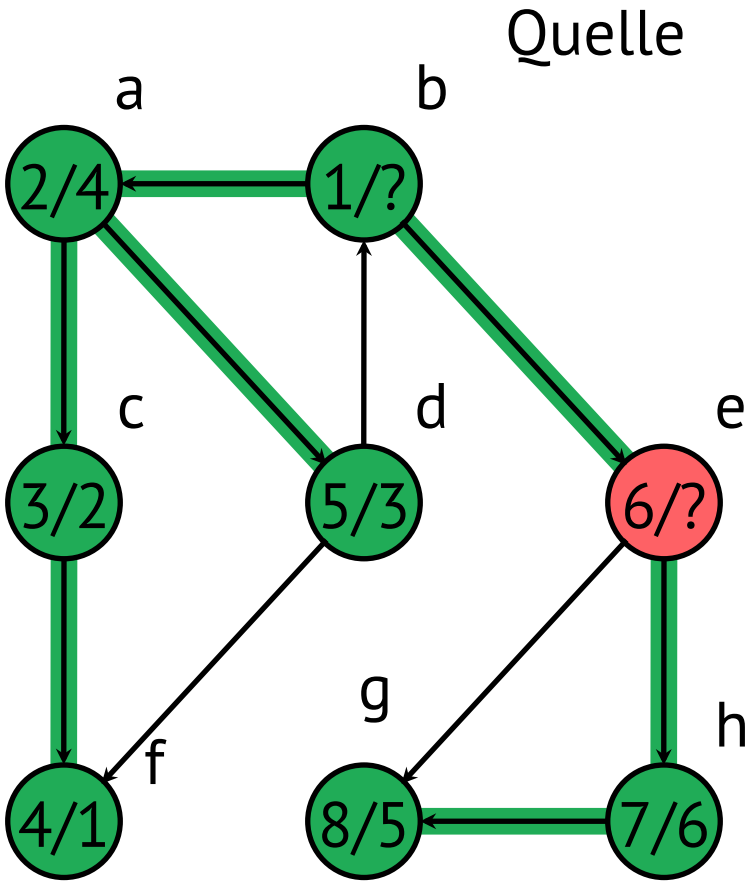
Beispiel



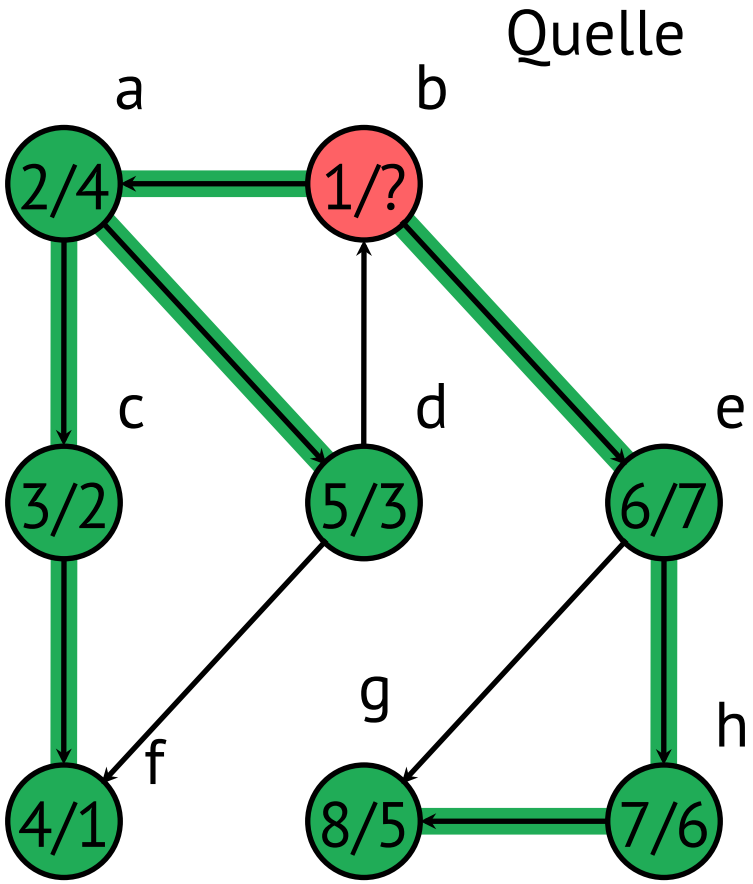
Beispiel



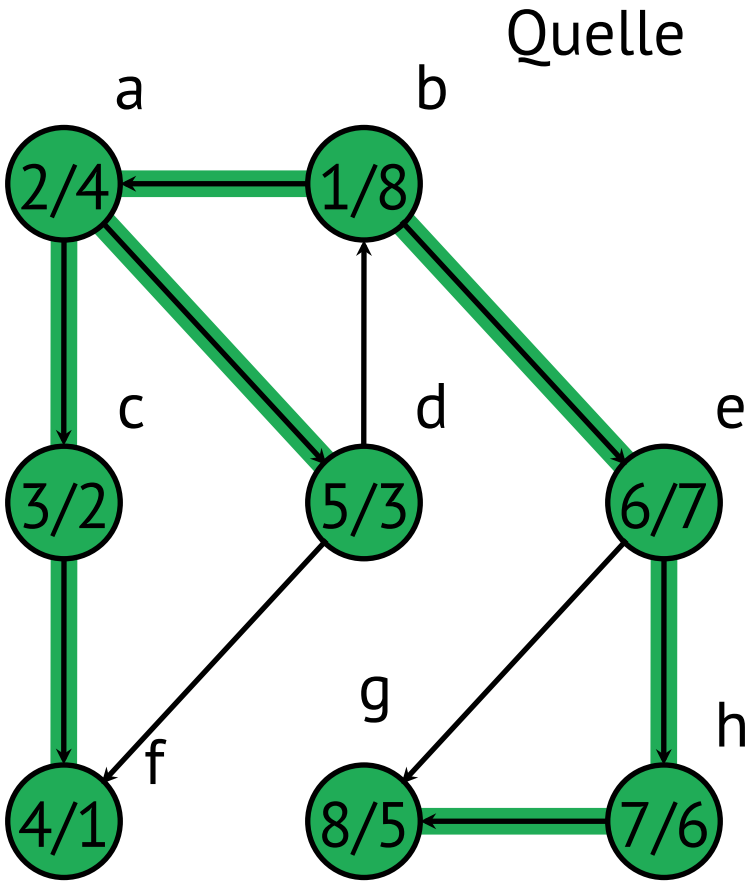
Beispiel



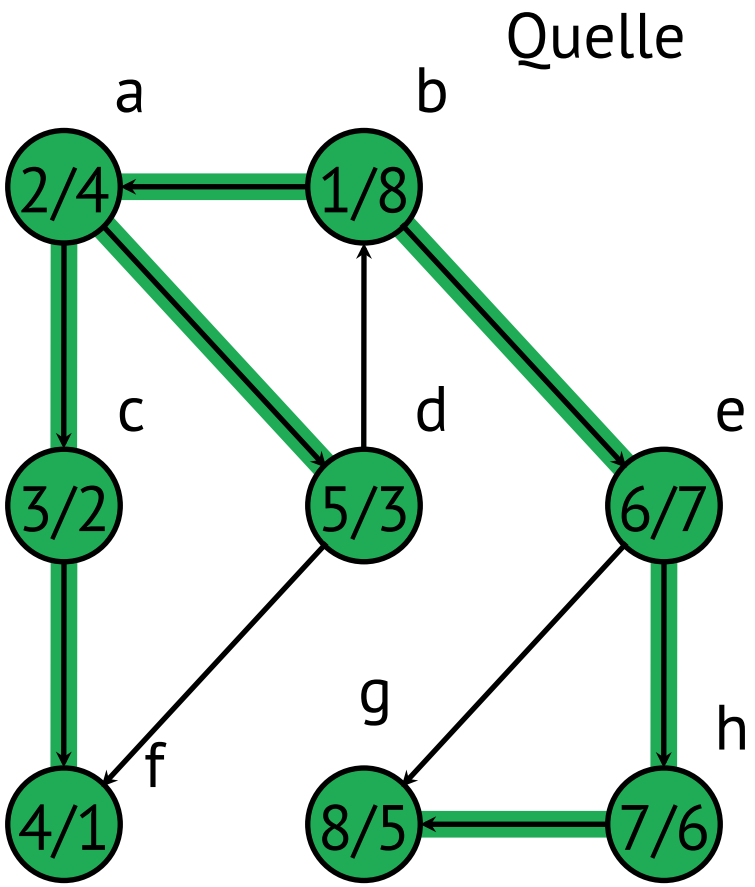
Beispiel

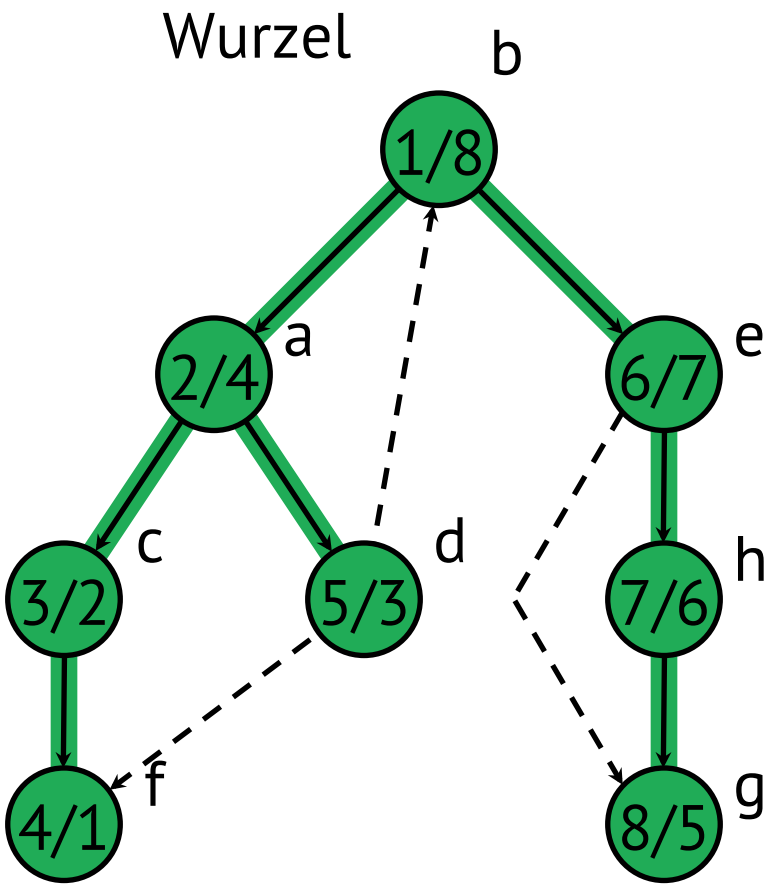


Beispiel

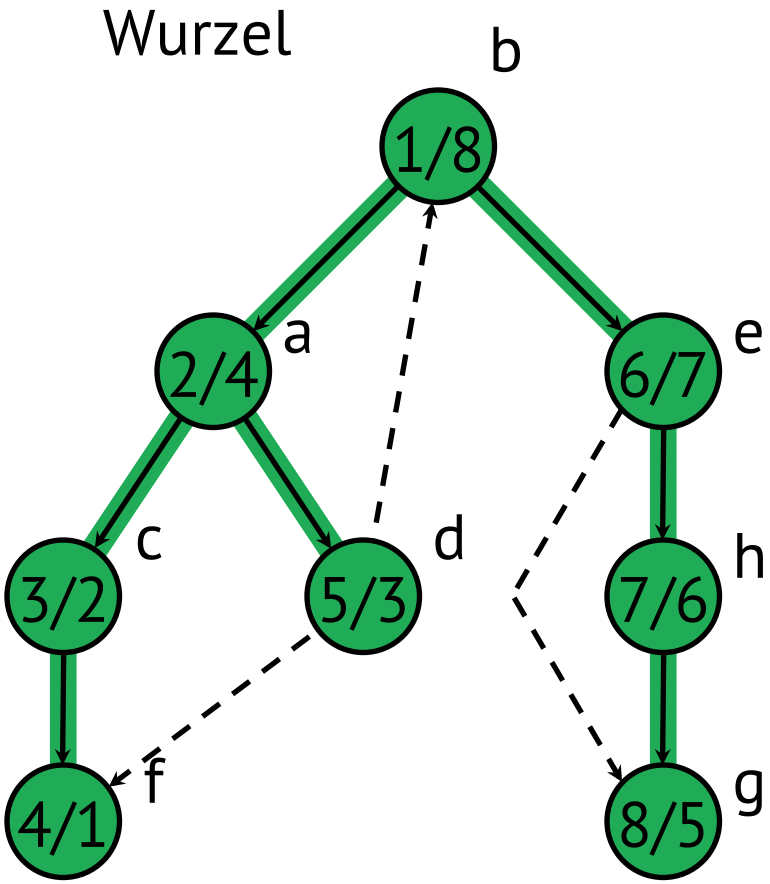


Tiefensuchbaum



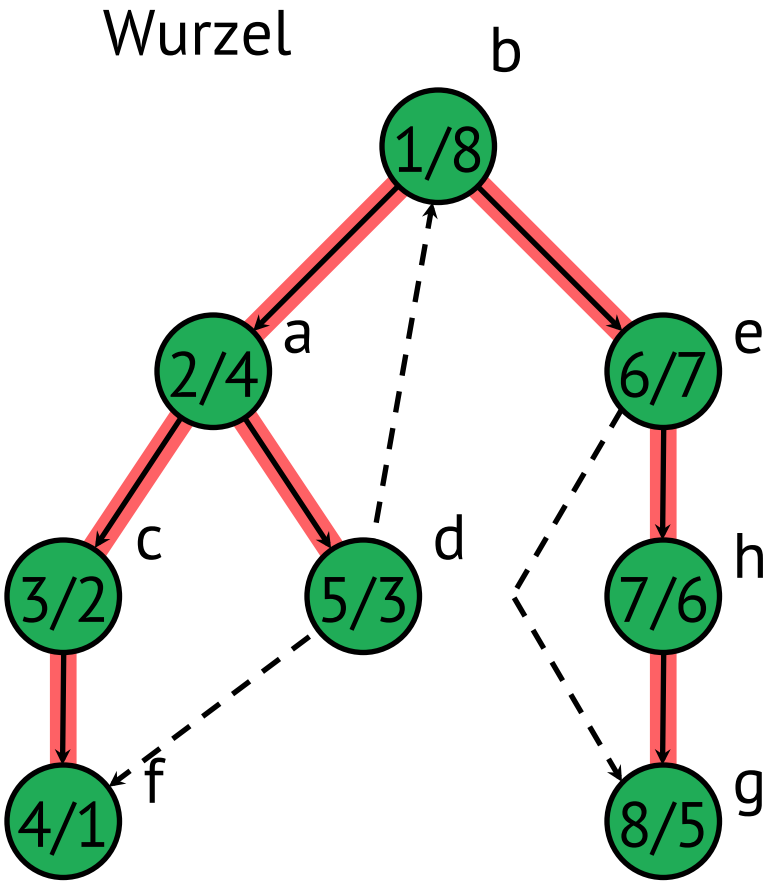


Tiefensuchbaum

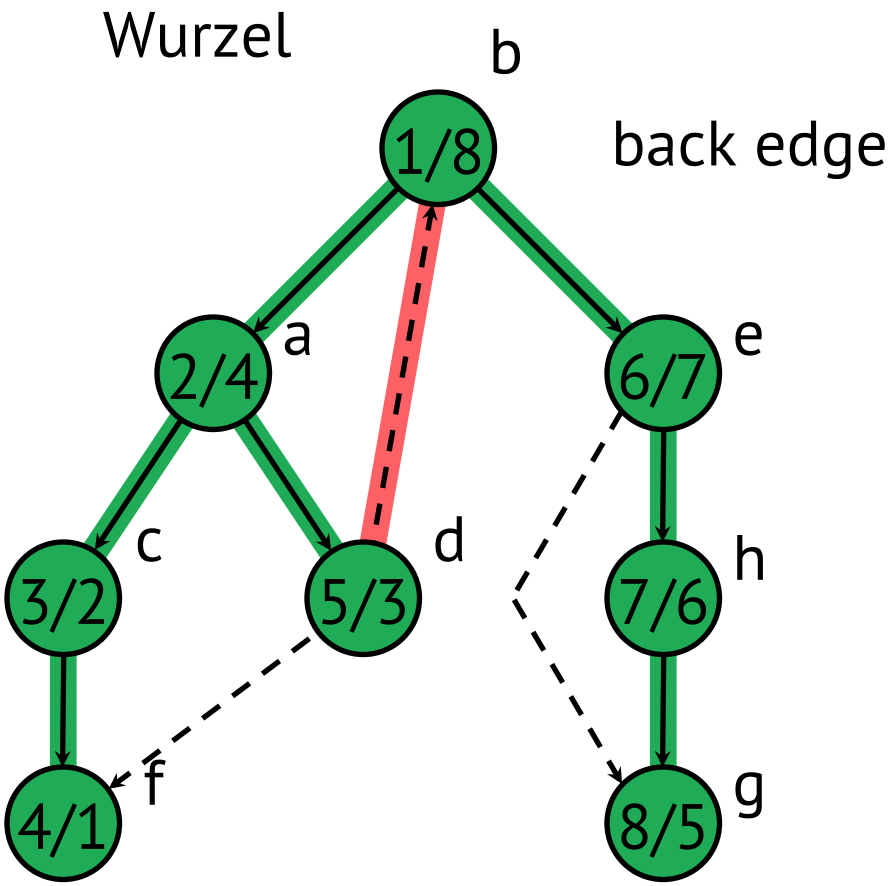


Tiefensuchbaum

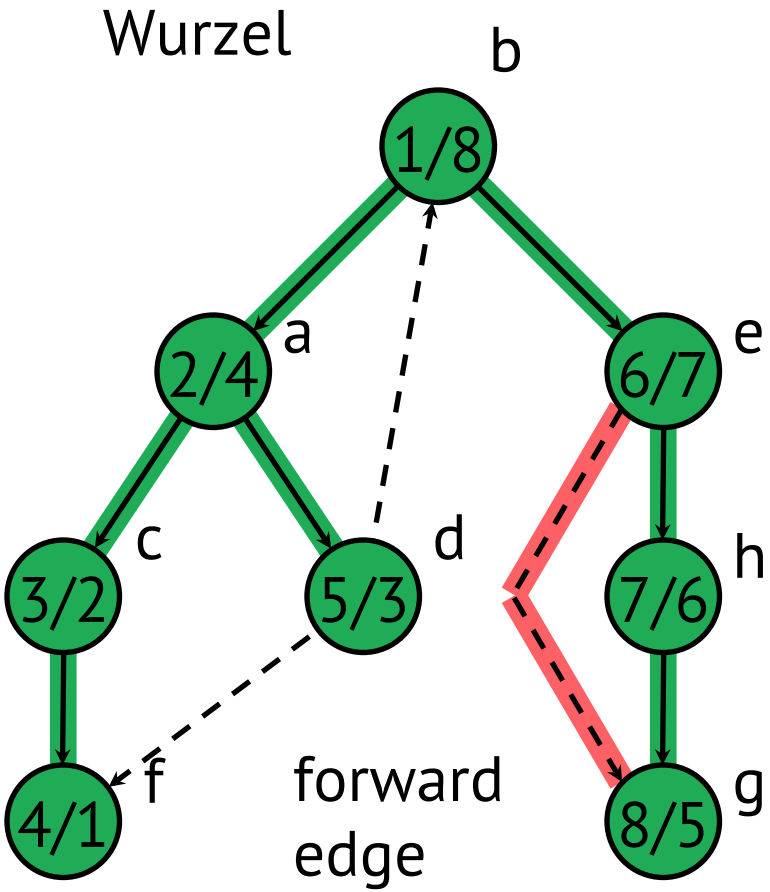
tree edges



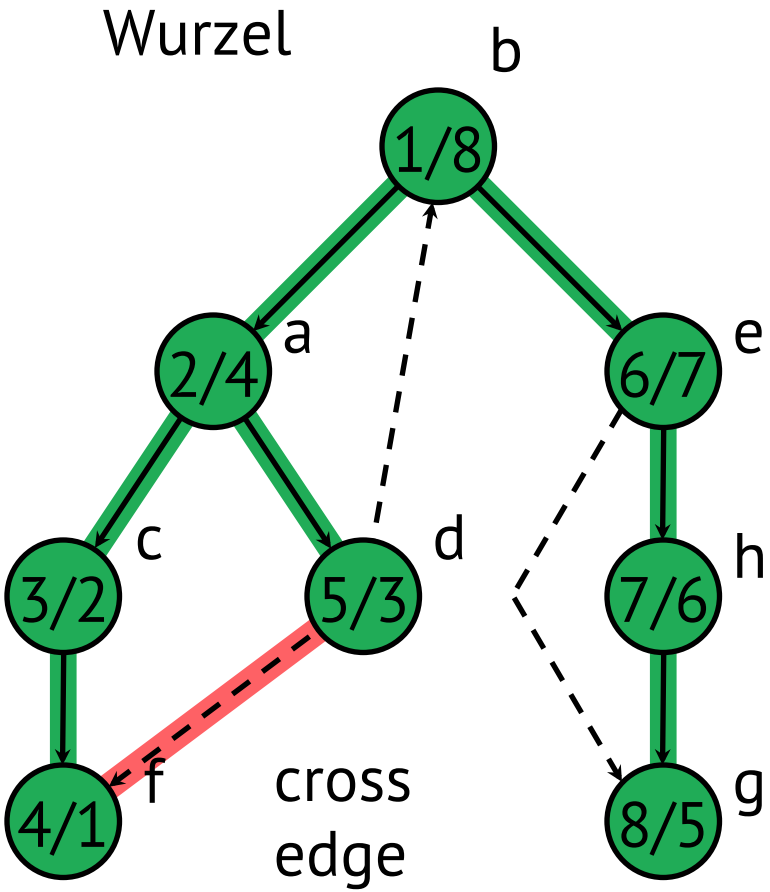
Tiefensuchbaum



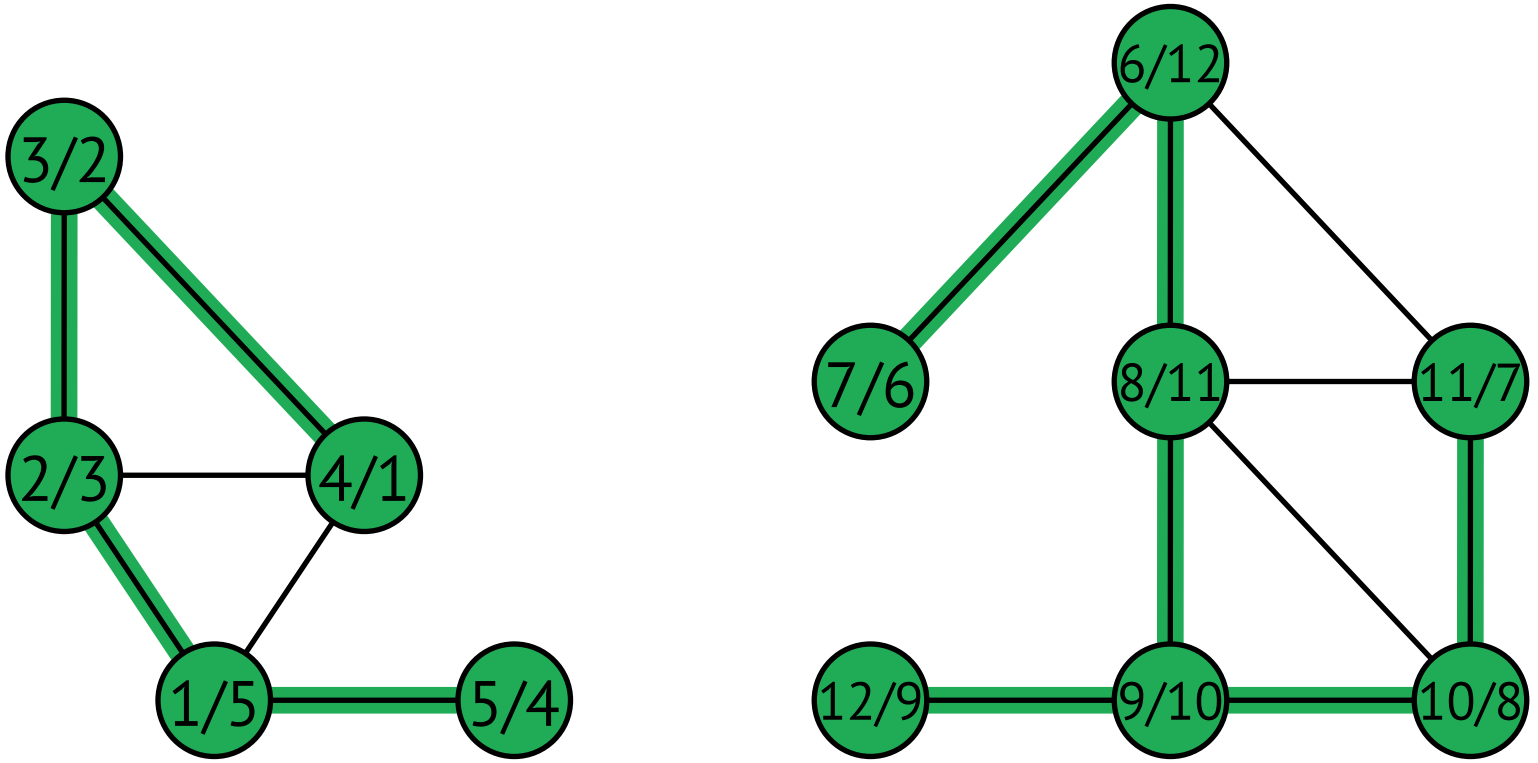
Tiefensuchbaum



Tiefensuchbaum



Tiefensuchwald



Tiefensuche

- DepthFirstSearch(G)
  for all v ∈ V do
    pre[v] ← 0
    post[v] ← 0
  prectr ← 0
  postctr ← 0
  for all v ∈ V do
    if pre[v] = 0 then
      DepthFirstVisit(v)

Tiefensuche

- DepthFirstVisit(u)
  - prectr ← prectr + 1
  - pre[u] ← prectr
  - for all v∈Adj[u] do
    - if pre[v] = 0 then
      - DepthFirstVisit(v)
  - postctr ← postctr + 1
  - post[u] ← postctr

Analyse

- DepthFirstSearch(G)

```
for all v ∈ V do
    pre[ v ] ← 0
    post[ v ] ← 0
prectr ← 0
postctr ← 0
for all v ∈ V do
    if pre[v] = 0 then
        DepthFirstVisit( v )
```

- Aufwand: O(V)

Analyse

- DepthFirstVisit(u)

```
    prectr ← prectr + 1
    pre[u] ← prectr
    for all v∈Adj[u] do
        if pre[v] = 0 then
            DepthFirstVisit( v )
    postctr ← postctr + 1
    post[u] ← postctr
```

- Aufwand: ...

Analyse

- Jeder Knoten wird genau einmal besucht.
- Adj[u] wird genau dann abgearbeitet, wenn u besucht wird.
- Der Aufwand zur Abarbeitung aller Adjazenzlisten ist also $\sum_{u \in V} |Adj[u]| = O(E)$

Analyse

- Gesamtaufwand der Tiefensuche

$$O(V+E)$$

2.6.2 Ausspähen von Graphen

2.6.2.1 Breitensuche

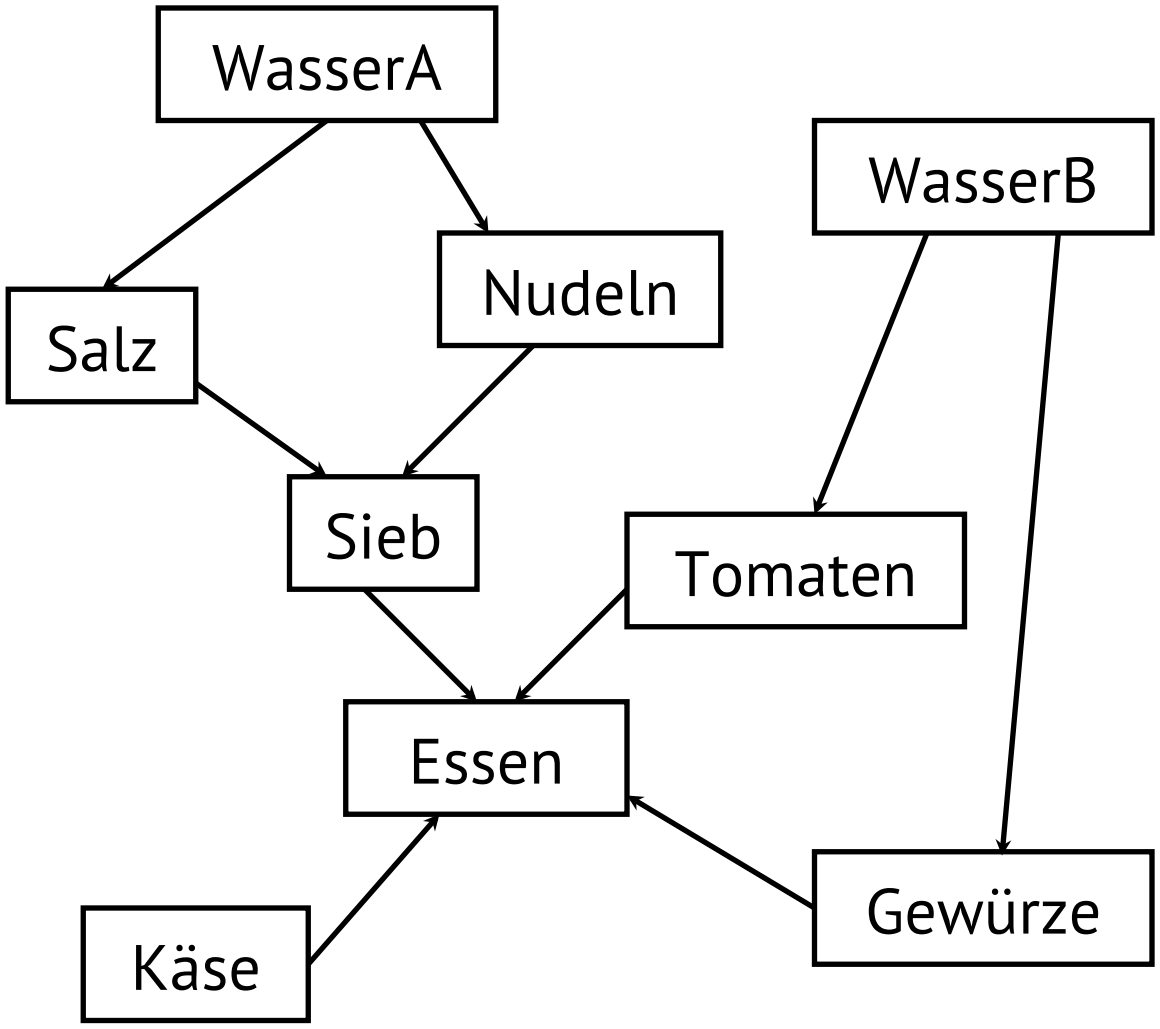
2.6.2.2 Tiefensuche

2.6.2.3 Topologisches Sortieren

Topologisches Sortieren

- Eine vollständige Ordnung $(V,<)$ heißt eine topologische Sortierung des gerichteten, zyklenfreien Graphen $G=(V,E)$, falls gilt $(u,v) \in E \rightarrow u < v$

Topologisches Sortieren

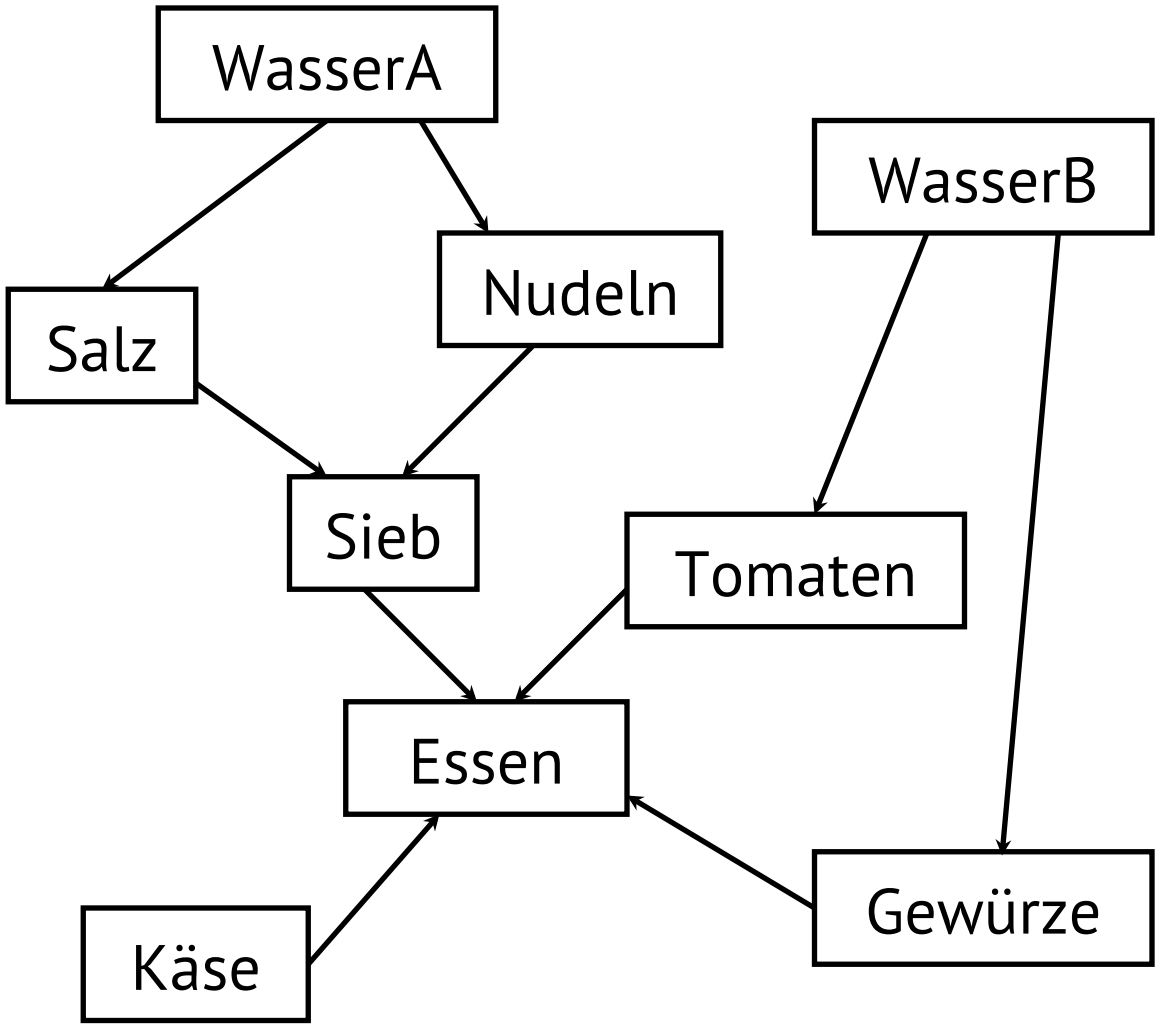


1	Käse
2	WasserB
3	Gewürze
4	Tomaten
5	WasserA
6	Nudeln
7	Salz
8	Sieb
9	Essen

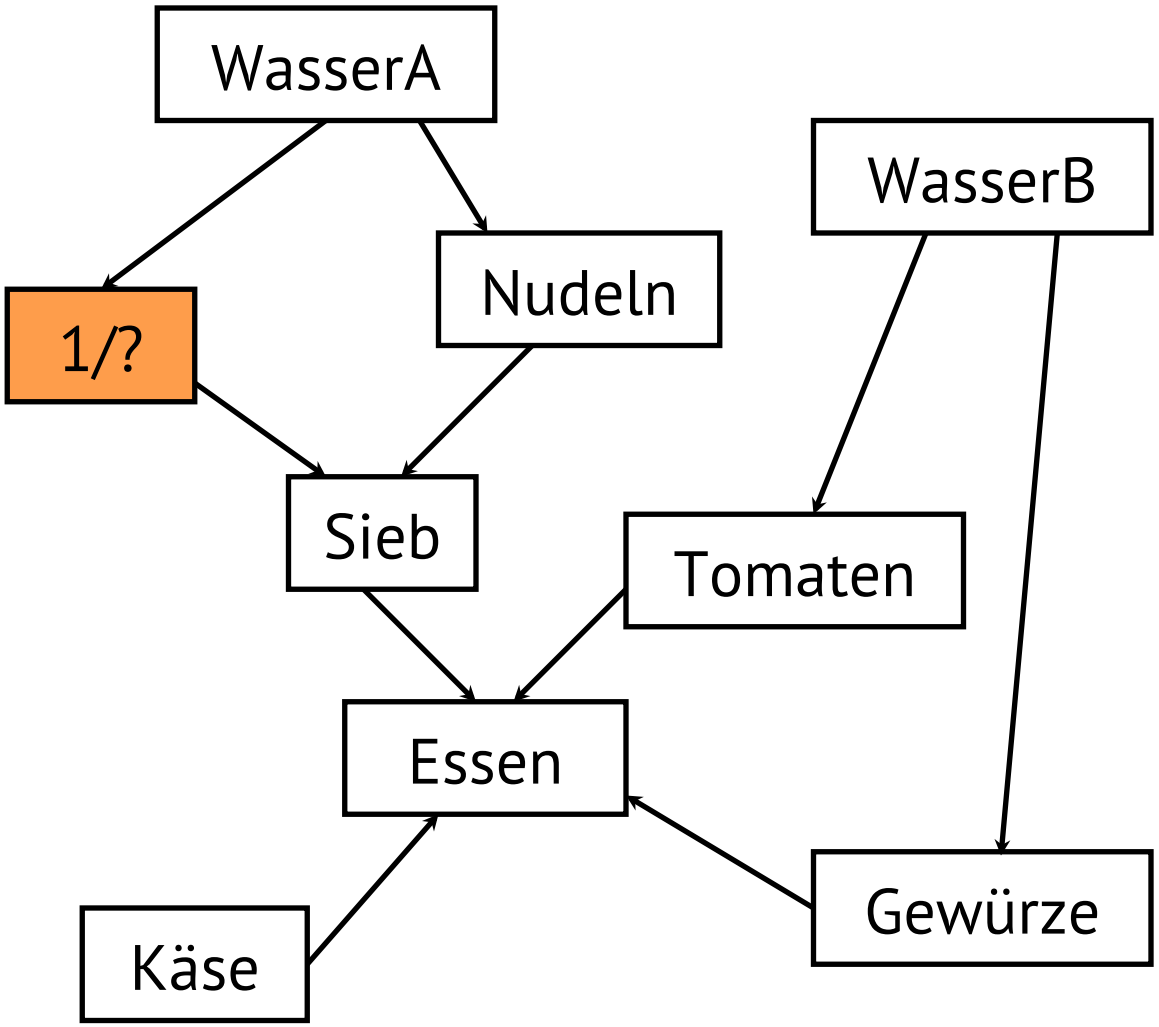
Topologisches Sortieren

- Es sei p die Postfixnummerierung des Tiefensuchwalds von G. Dann ist die Ordnung der Knoten v nach absteigendem p(v) eine topologische Sortierung von G.

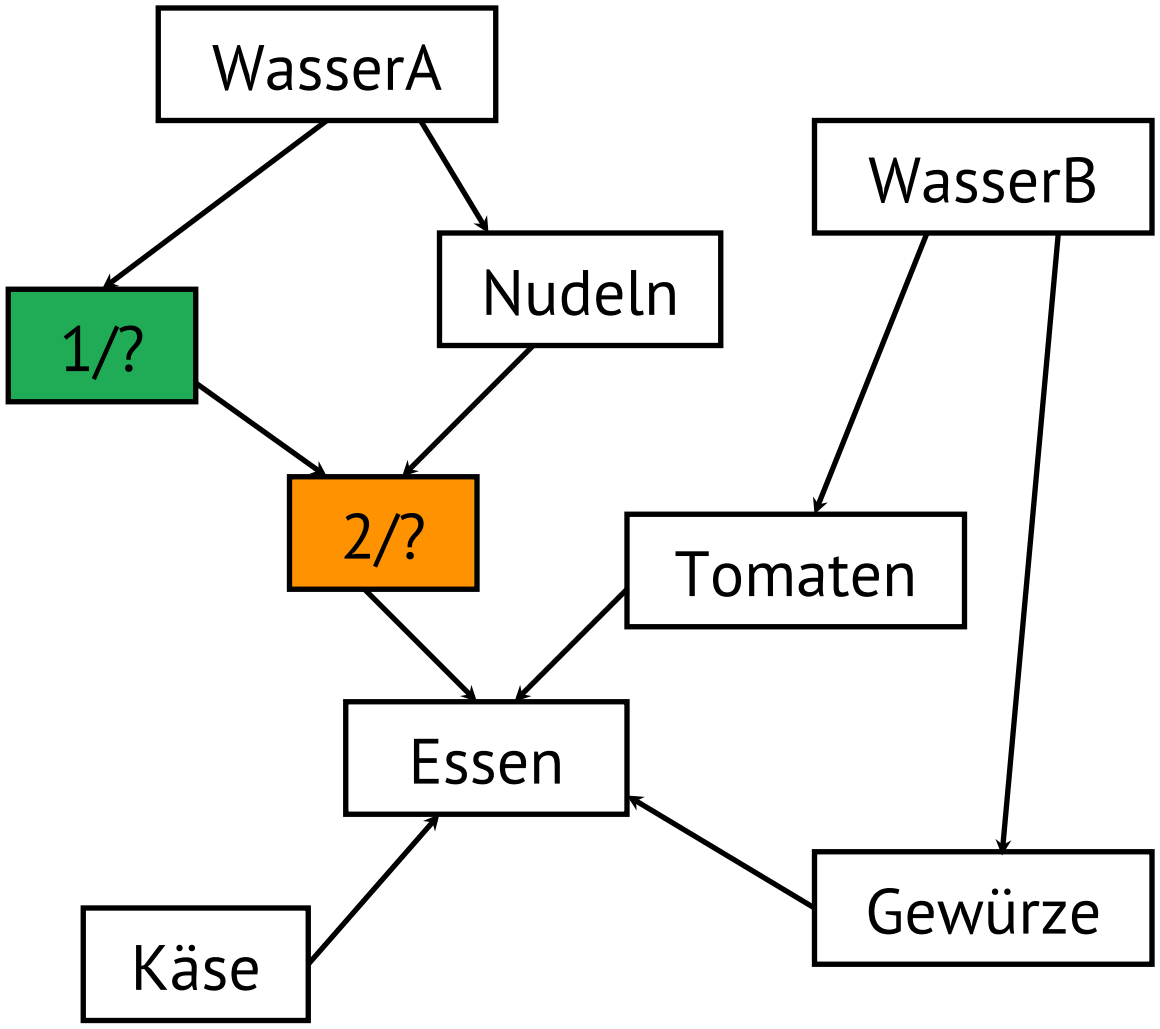
Topologisches Sortieren



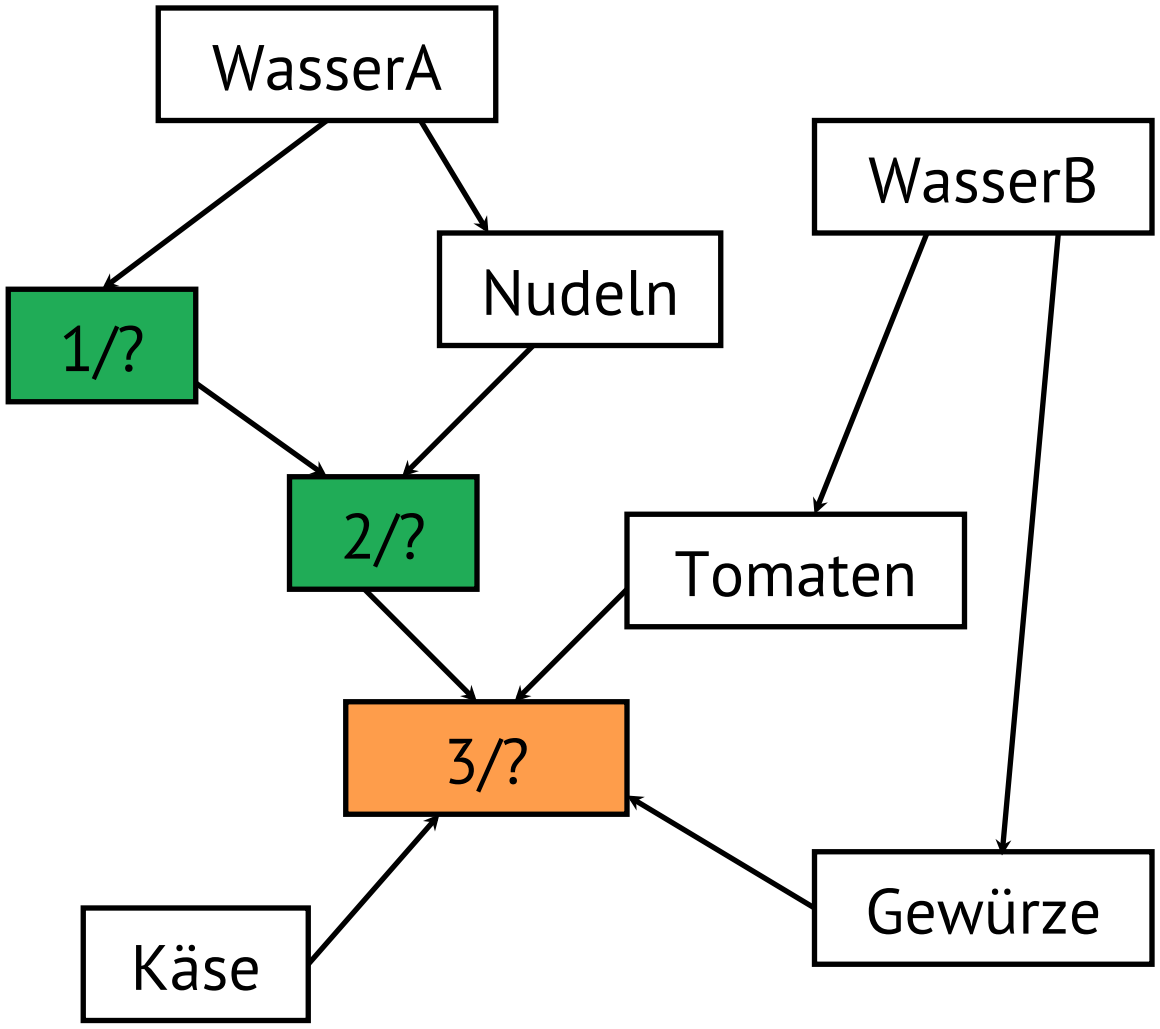
Topologisches Sortieren



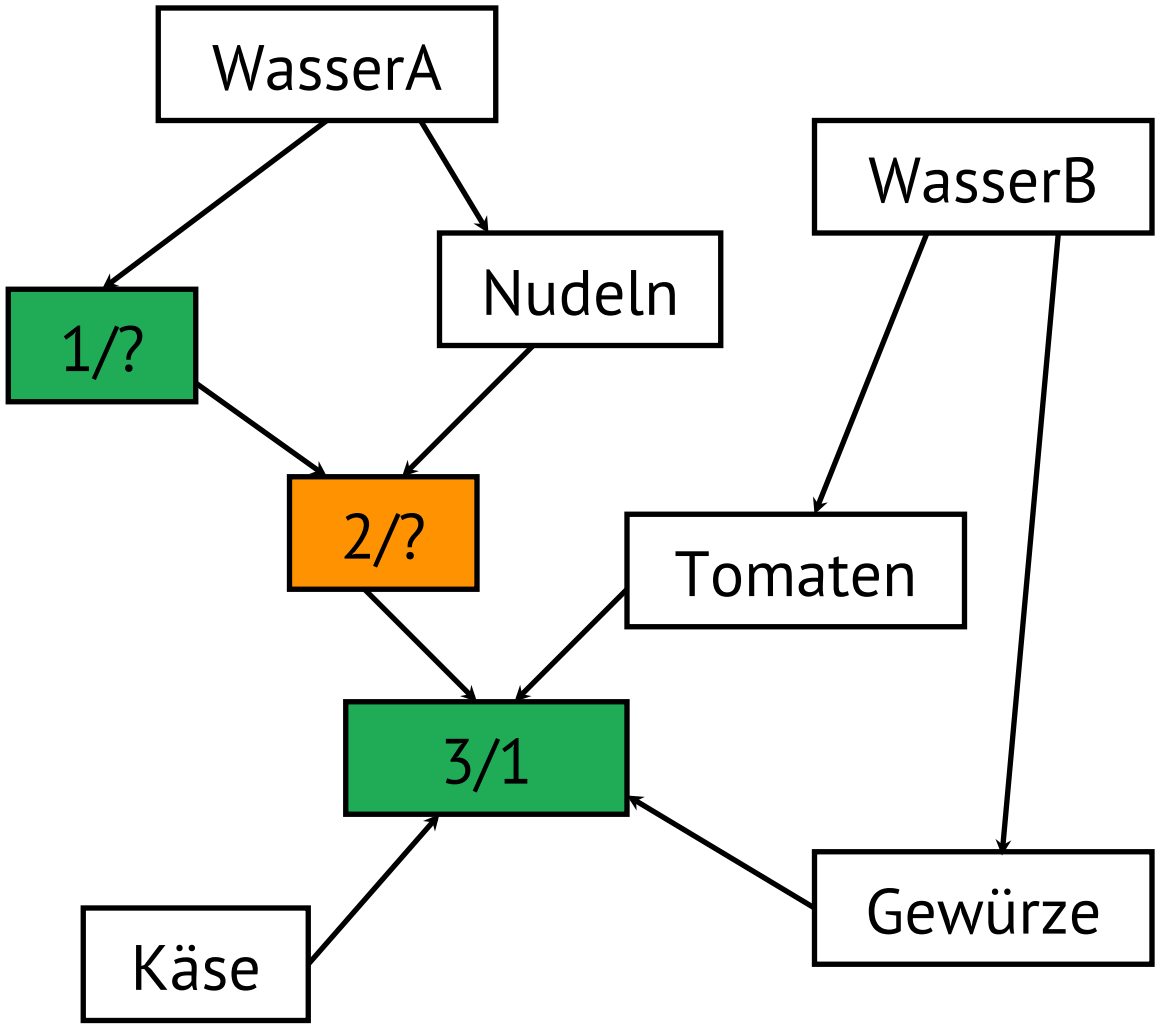
Topologisches Sortieren



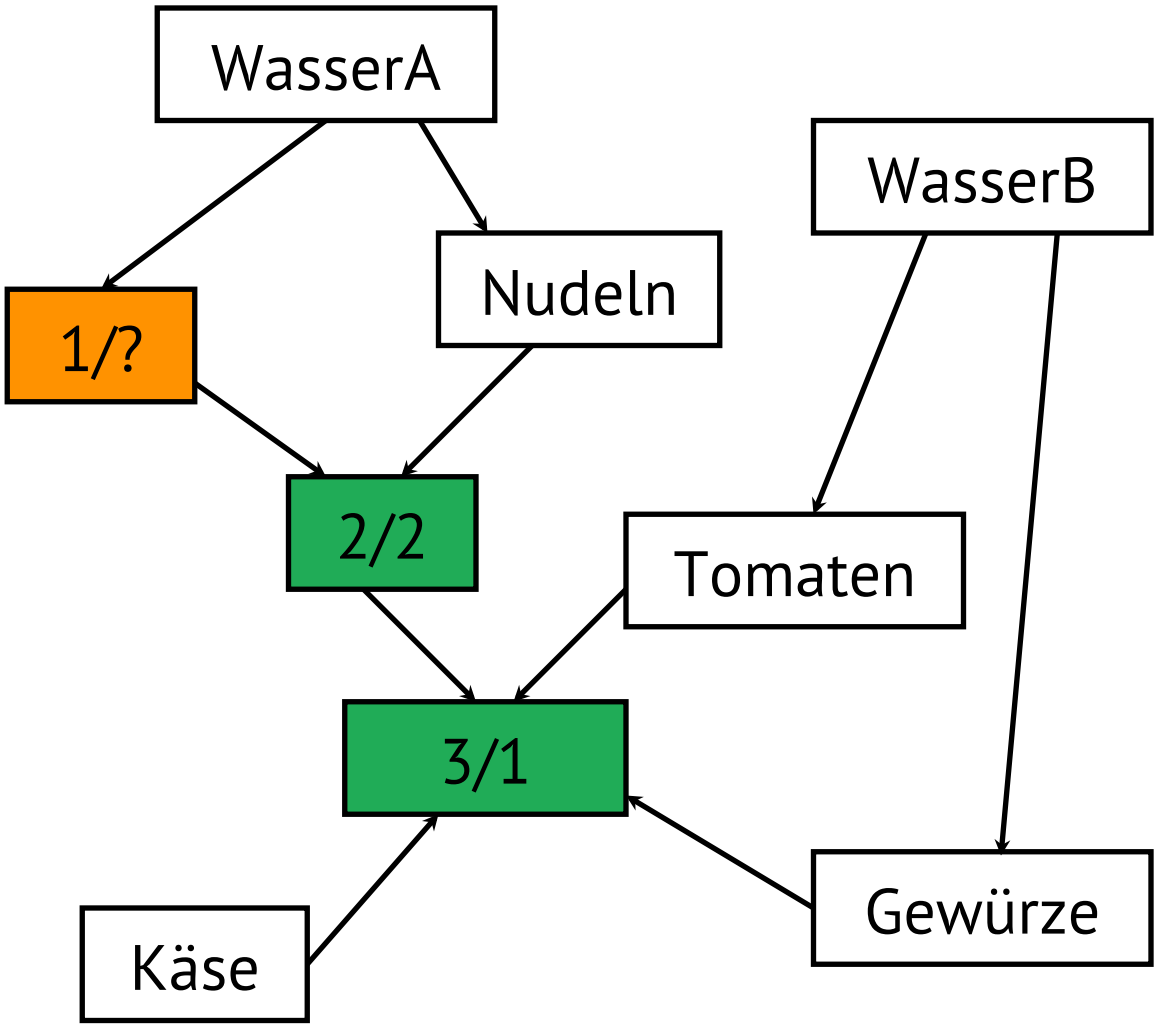
Topologisches Sortieren



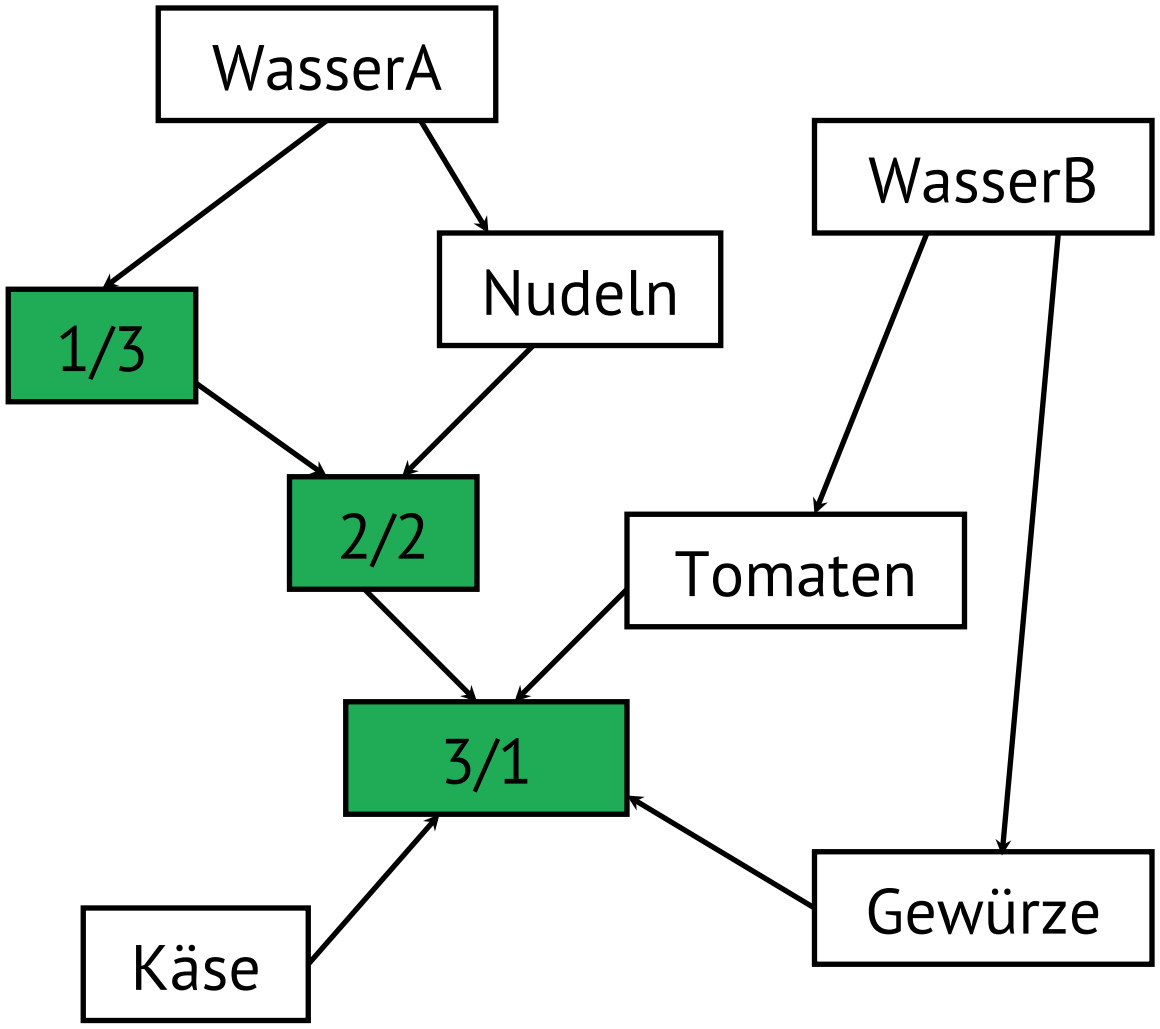
Topologisches Sortieren



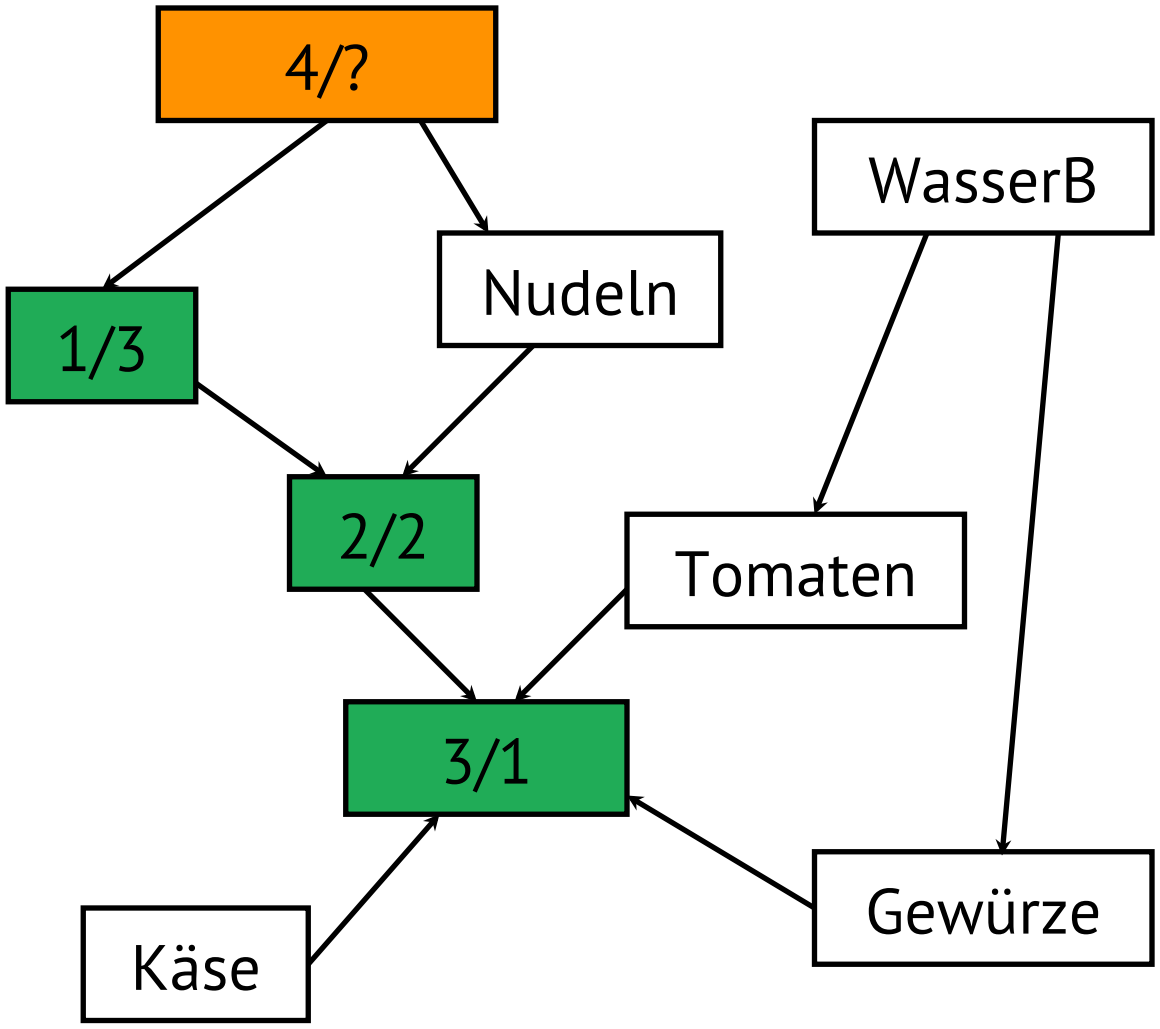
Topologisches Sortieren



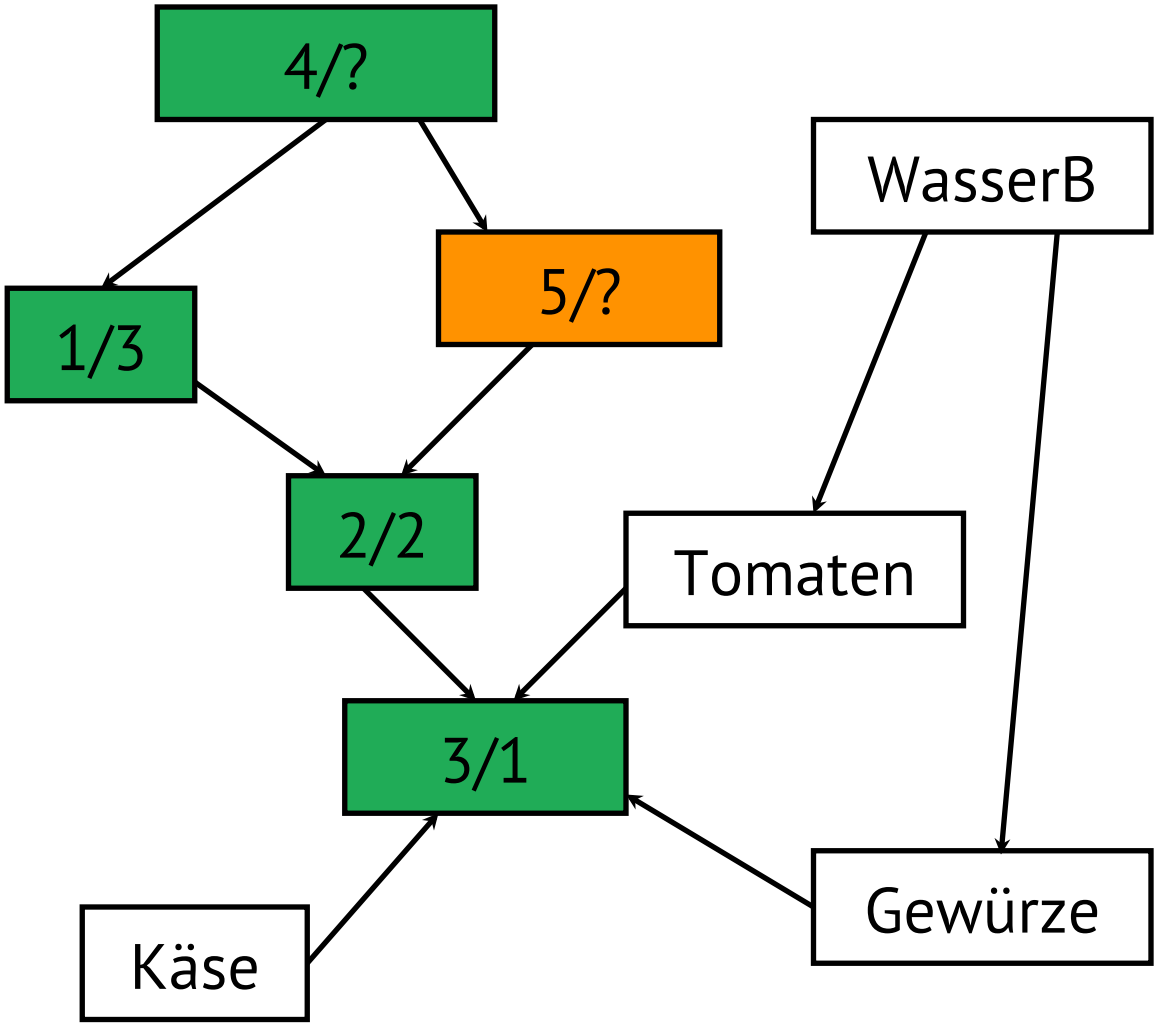
Topologisches Sortieren



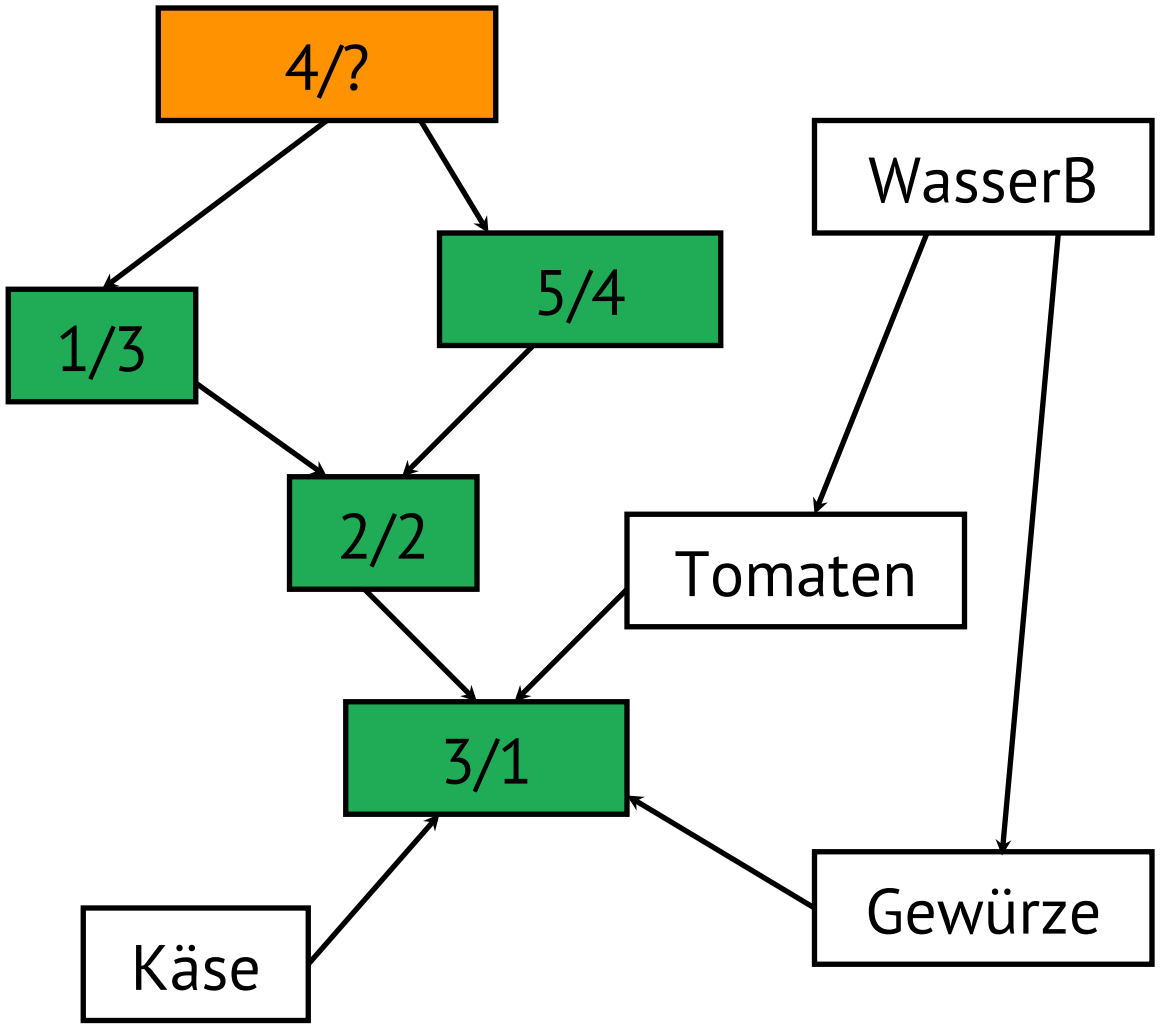
Topologisches Sortieren



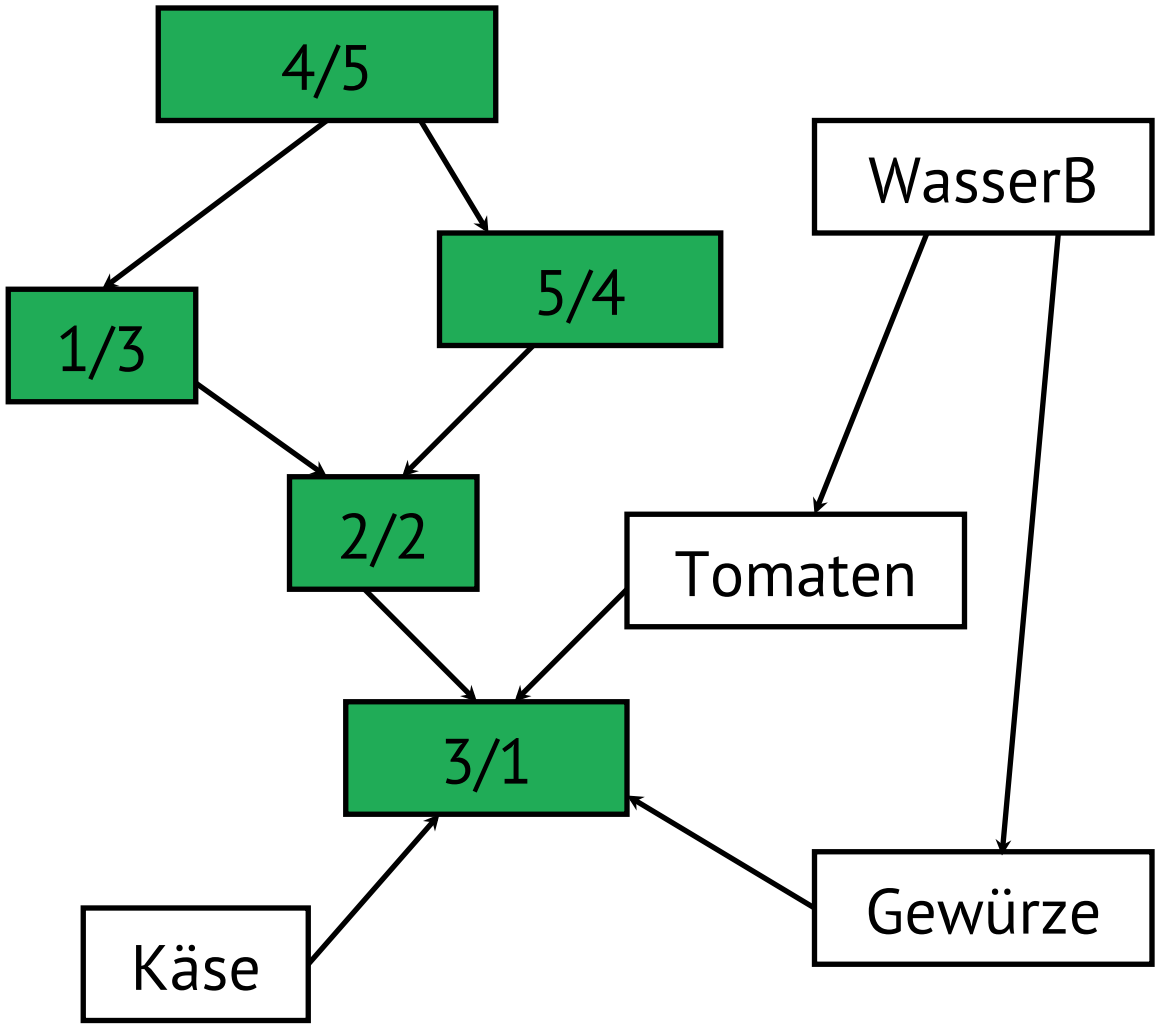
Topologisches Sortieren



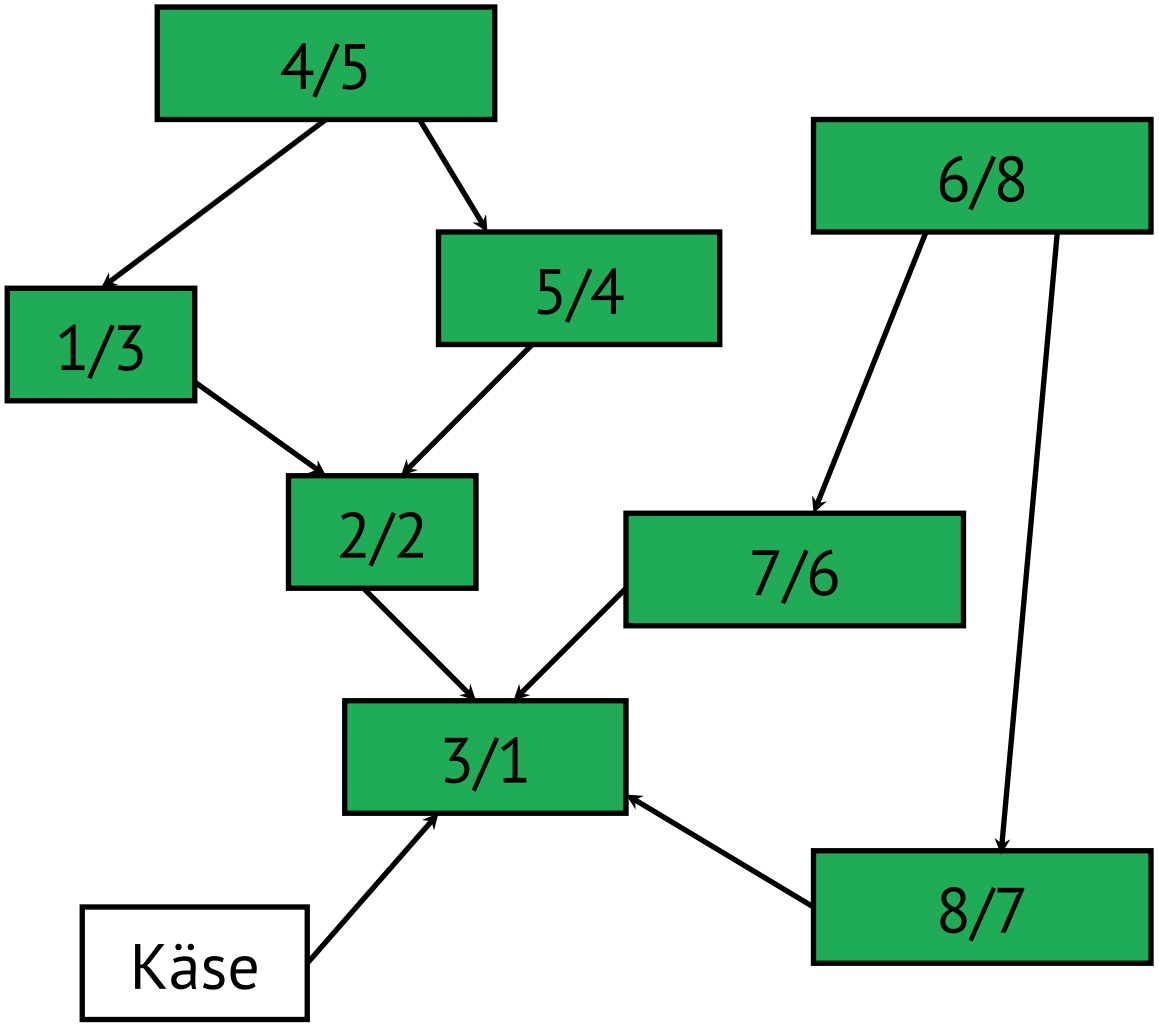
Topologisches Sortieren



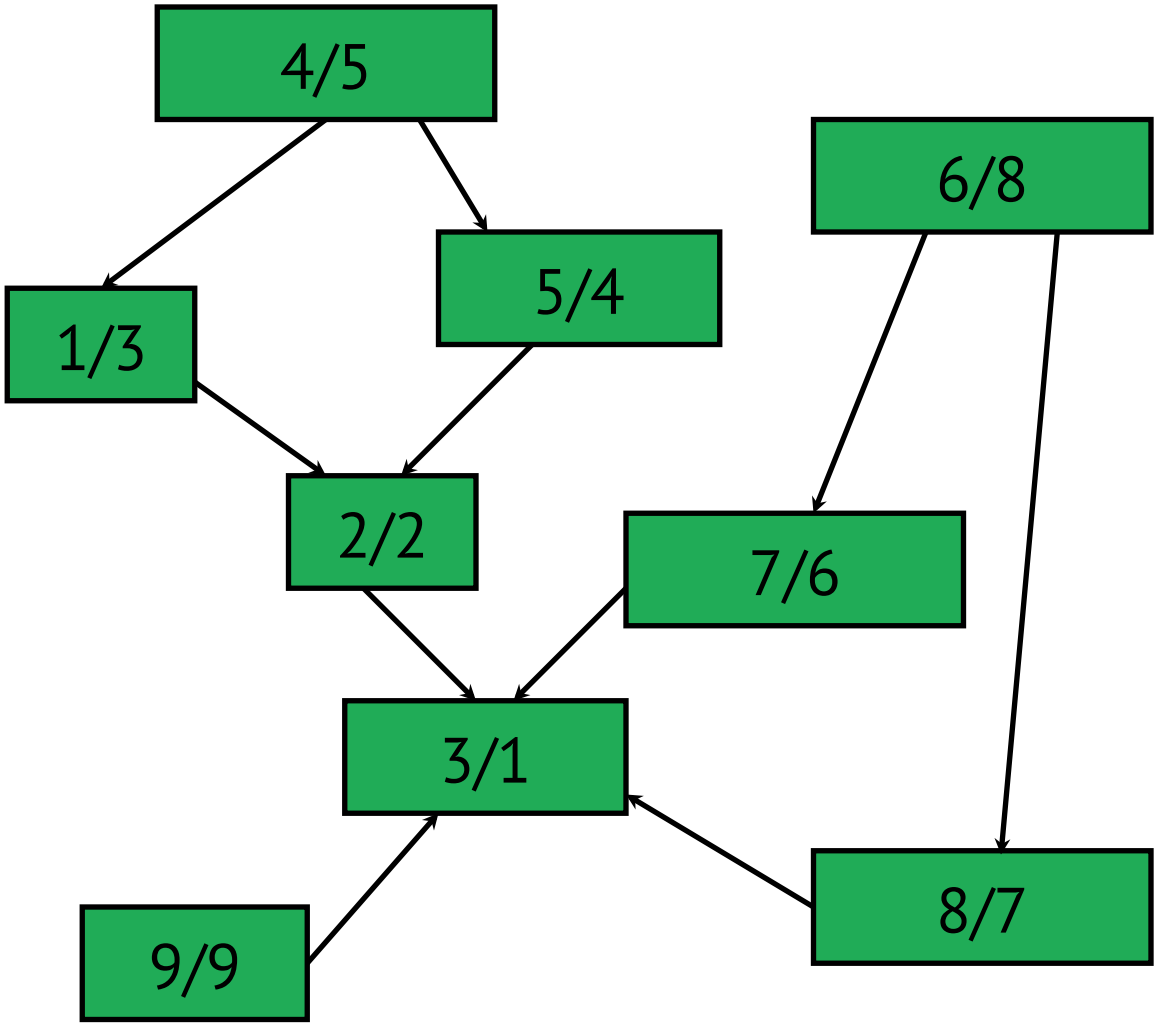
Topologisches Sortieren



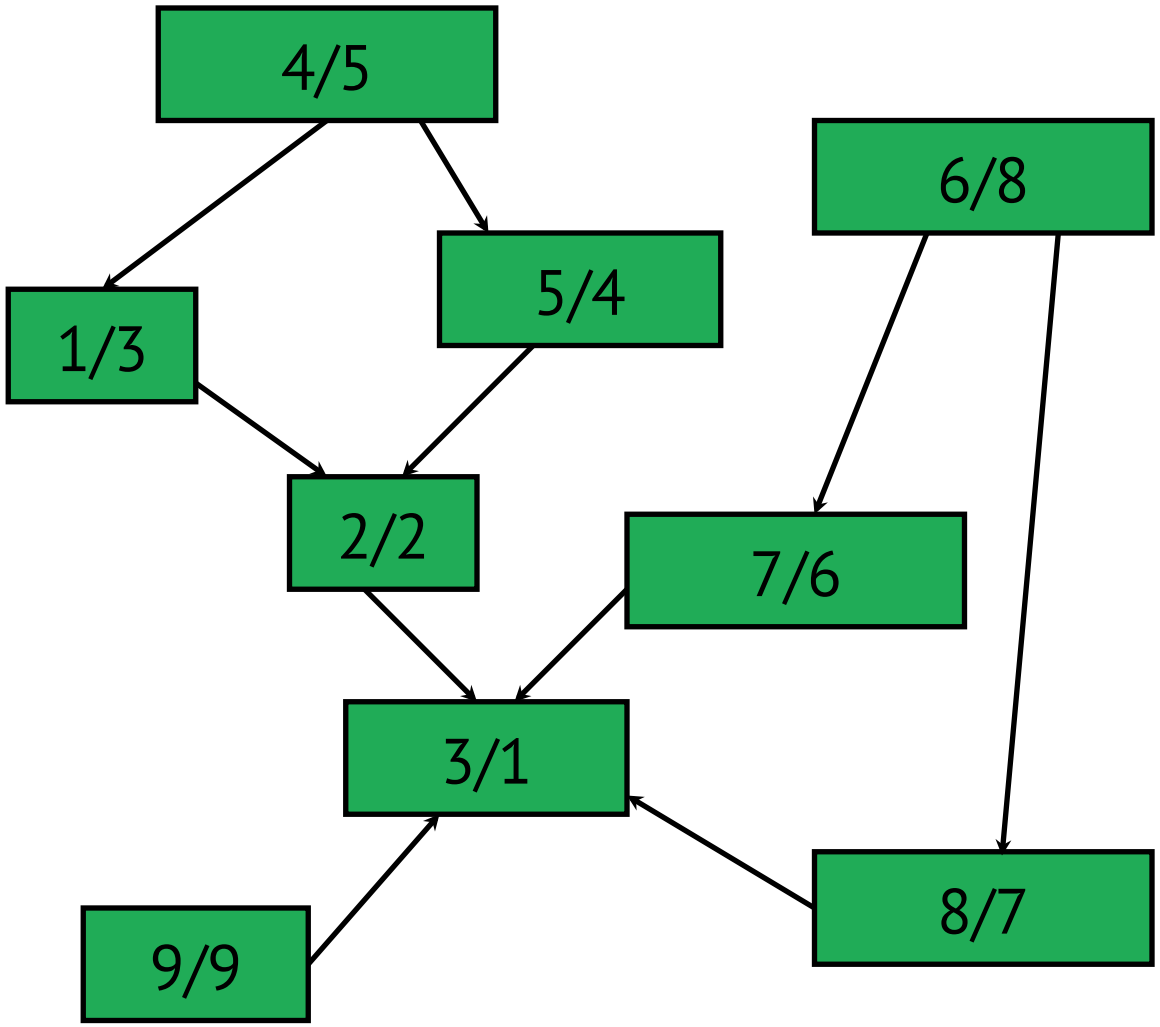
Topologisches Sortieren



Topologisches Sortieren

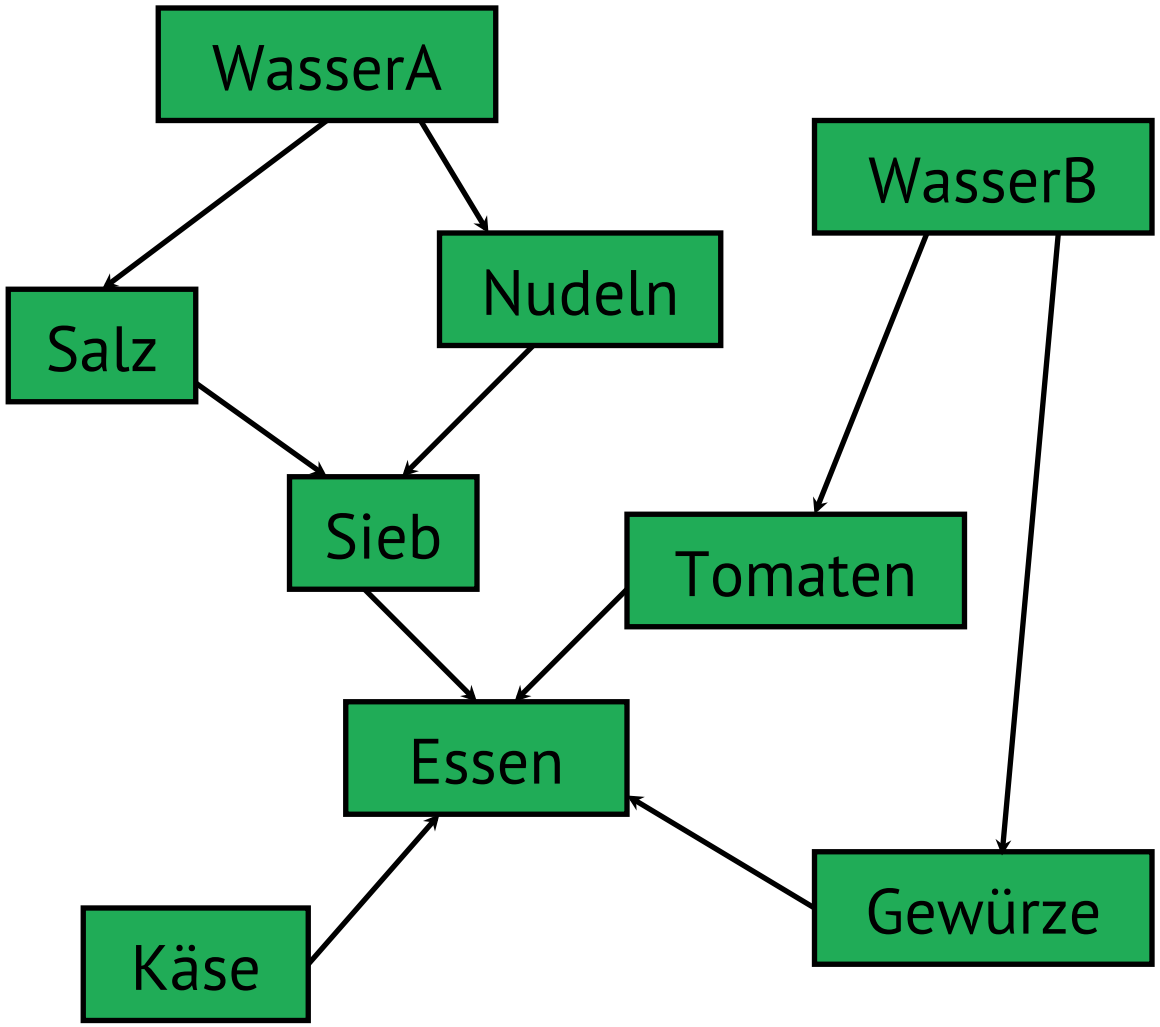


Topologisches Sortieren



1	9/9
2	6/8
3	8/7
4	7/6
5	4/5
6	5/4
7	1/3
8	2/2
9	3/1

Topologisches Sortieren



1	Käse
2	WasserB
3	Gewürze
4	Tomaten
5	WasserA
6	Nudeln
7	Salz
8	Sieb
9	Essen